Have the probability that the selected number would be divisible by 4 or 7 is \(\frac{9}{25}\) or 0.36

**Example 16.10:** A coin is tossed thrice. What is the probability of getting 2 or more heads?

**Solution:** If a coin is tossed three times, then we have the following sample space. 

\[ S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\} \]

2 or more heads imply 2 or 3 heads. If \(A\) and \(B\) denote the events of occurrence of 2 and 3 heads respectively, then we find that 

\[ A = \{HHT, HTH, THH\} \quad \text{and} \quad B = \{HHH\} \]

\[ \therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{8} \]

and 

\[ P(B) = \frac{n(B)}{n(S)} = \frac{1}{8} \]

As \(A\) and \(B\) are mutually exclusive, the probability of getting 2 or more heads is

\[ P(A \cup B) = P(A) + P(B) \]

\[ = \frac{3}{8} + \frac{1}{8} \]

\[ = 0.50 \]

**Example 16.11:** A number is selected at random from the first 1000 natural numbers. What is the probability that it would be a multiple of 5 or 9?

**Solution:** Let \(A\), \(B\), \(A \cup B\) and \(A \cap B\) denote the events that the selected number would be a multiple of 5, 9, 5 or 9 and both 5 and 9 i.e. LCM of 5 and 9 i.e. 45 respectively.
Since \(1000 = 5 \times 200\)  
\[= 9 \times 111 + 1\]  
\[= 45 \times 22 + 10,\]
it is obvious that  
\[P(A) = \frac{200}{1000}, P(B) = \frac{111}{1000}, P(A \cap B) = \frac{22}{1000}\]

Hence the probability that the selected number would be a multiple of 4 or 9 in given by  
\[P(A \cup B) = P(A) + P(B) - P(A \cap B)\]
\[= \frac{200}{1000} + \frac{111}{1000} - \frac{22}{1000}\]
\[= 0.29\]

**Example 16.12:** The probability that an Accountant’s job applicant has a B. Com. Degree is 0.85, that he is a CA is 0.30 and that he is both B. Com. and CA is 0.25 out of 500 applicants, how many would be B. Com. or CA?

**Solution:** Let the event that the applicant is a B. Com. be denoted by B and that he is a CA be denoted by C Then as given,

\[P(B) = 0.85, P(C) = 0.30\]  
and \(P(B \cap C) = 0.25\)

The probability that an applicant is B. Com. or CA is given by  
\[P(B \cup C) = P(B) + P(C) - P(B \cap C)\]
\[= 0.85 + 0.30 - 0.25\]
\[= 0.90\]

**Example 16.13:** If \(P(A-B) = \frac{1}{5}\), \(P(A) = \frac{1}{3}\) and \(P(B) = \frac{1}{2}\), what is the probability that out of the two events A and B, only B would occur?

**Solution:** A glance at Figure 13.3 suggests that  
\[P(A-B) = P(A \cap B') = P(A) - P(A \cap B) \quad (13.21)\]

And  
\[P(B-A) = P(B \cap A') = P(B) - P(A \cap B) \quad (13.22)\]

Also (13.21) and (13.22) describe the probabilities of occurrence of the event only A and only B respectively.

As given \(P(A-B) = \frac{1}{5}\)

\[\Rightarrow P(A) - P(A \cap B) = \frac{1}{5}\]
\[
\Rightarrow \frac{1}{3} - P(A \cap B) = \frac{1}{5} \quad \text{[Since } P(A) = 1/3]\]

\[
\Rightarrow P(A \cap B) = \frac{2}{15}
\]

The probability that the event B only would occur

\[
= P(B - A)
\]

\[
= P(B) - P(A \cap B)
\]

\[
= \frac{1}{2} - \frac{2}{15} \quad \text{[Since } P(B) = \frac{1}{2}]\]

\[
= \frac{11}{30}
\]

**Example 16.14:** There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years is 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

**Solution**

As given \(P(A) = 0.80, P(B) = 0.60, P(C) = 0.50,\)

\(P(A \cap B) = 0.46, P(B \cap C) = 0.32, P(A \cap C) = 0.48\) and

\(P(A \cap B \cap C) = 0.26\)

The probability that at least one of them survives another 5 years in given by

\[
P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \quad \text{[Equation (16.23)]}
\]

\[
= 0.80 + 0.60 + 0.50 - 0.46 - 0.32 - 0.48 + 0.26
\]

\[
= 0.90
\]
मिश्रित संभावना या संयुक्त संभावना (Compound Probability or Joint Probability)

एक घटना की संभावना, अभी तक समझाई जा चुकी है, को तकनीकी तौर पर सरल हीन या सीमान्त संभावना माना जाता है।
लेकिन ऐसी भी परिस्थितियाँ होती हैं जो एक से अधिक घटना की आवृत्ति की संभावना की माप नहीं करती हैं। दो घटनाओं A तथा B को साथ-साथ आवृत्ति की संभावना को मिश्रित संभावना या घटनाओं को संयुक्त संभावना के रूप में जाना जाता है।
A तथा B को व्यक्त किया जाता है P(A∩B) तो इसी तरीके से K घटनाओं A1, A2, ...AK को साथ-साथ आवृत्ति की संभावना को व्यक्त किया जाता है P(A1∩A2∩...∩AK)

do घटनाओं A तथा B की मिश्रित संभावना के मामले में, हम दो परिस्थितियों का समान करते हैं। पहले मामले में, यदि एक घटना की आवृत्ति, जैसे B, एक अन्य घटना A की आवृत्ति द्वारा प्रभावित होती है तो दोनों घटनाओं A तथा B को आवृत्ति घटनाओं के रूप में जाना जाता है। हम P(B/A) उल्लेख का प्रयोग करते हैं, जिसको 'घटना B की संभावना, दिया गया है कि घटना A के प्रति तकनीक सुझाव जा सके कि एक अन्य घटना B घटित होगी यदि तथा केवल यदि पहली घटना A पहले ही घटित हो चुकी है। यह निम्न द्वारा दिया जायेगा--

\[
P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}
\]

(16.24)

वशतः कि P(A) > 0 अर्थात् A एक अस्वभाव घटना नहीं है।

Similarly, P(A/B) \neq \frac{P(A \cap B)}{P(B)} .... (16.25)

if P(B) > 0.

एक उदाहरण के रूप में यदि एक बक्स में 5 लाल तथा 8 सफेद गेंदें हैं तथा उसमें से 2 गेंदें को दो क्रमशः छुड़ा किये जाते हैं, तब A उनमें लाल की संभावना कि छुड़ी गेंद लाल से 2 लाल गेंदें लें। आवृत्ति P(A) = \frac{5}{13} \times \frac{4}{12} = \frac{5}{39}

सर्व संभावना का एक उदाहरण है चूँकि ऐसा निकाला जाना दिया गया है कि पहले छुड़ी लाल गेंदें हैं, बक्स में गेंदों का गुणधर्म बदल जाता है तथा इसलिए दूसरी छुड़ी 2 सफेद गेंदों की आवृत्ति (B2) पहले 2 सफेद गेंद (R2) के परिणाम पर निर्भर करती है। यह घटना निम्न द्वारा दी जा सकती है--

\[
P(B2/R2) = \frac{P(B2 \cap R2)}{P(R2)}
\]

(16.26)

इस परिस्थिति में, यदि दूसरी घटना B की आवृत्ति पहली घटना A की आवृत्ति से प्रभावित नहीं होती, तो B को A से स्वतंत्र माना जायेगा। यह भी बताता है कि इस दशा में, A भी B से स्वतंत्र हैं तथा A तथा B को परस्पर तीर पर स्वतंत्र माना जाता है या मात्र स्वतंत्र। इस दशा में हम पाते हैं--

\[
P(B/A) = P(B)
\]

(16.26)

and also P(A/B) = P(A)

(16.27)

There by implying, P(A \cap B) = P(A) \times P(B)

(16.28)

In the above example, if the balls are drawn with replacement, then the two events B2 and R2 are independent and we have
P(B_2 / R_2) = P(B_2)

(16.28) is the necessary and sufficient condition for the independent of two events. In a similar manner, three events A, B and C are known as independent if the following conditions hold:

\[
P(A \cap B) = P(A) \times P(B)
\]

\[
P(A \cap C) = P(A) \times P(C)
\]

\[
P(B \cap C) = P(B) \times P(C)
\]

\[
P(A \cap B \cap C) = P(A) \times P(B) \times P(C) \tag{16.29}
\]

It may be further noted that if two events A and B are independent, then the following pairs of events are also independent:

(i) A and B'
(ii) A' and B
(iii) A' and B'

**Theorems of Compound Probability**

Theorem 6: For any three events A, B and C, the probability that they occur jointly is given by

\[
P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/(A \cap B)) \tag{16.32}
\]

Provided P(A \cap B) > 0

In the event of independence of the events

(16.31) and (16.32) are reduced to

\[
P(A \cap B) = P(A) \times P(B)
\]

and

\[
P(A \cap B \cap C) = P(A) \times P(B) \times P(C)
\]

which we have already discussed.

**Example 16.15:** Rupesh is known to hit a target in 5 out of 9 shots whereas David is known to hit the same target in 6 out of 11 shots. What is the probability that the target would be hit once they both try?

**Solution:** Let A denote the event that Rupesh hits the target and B, the event that David hits the target. Then as given,

\[
P(A) = \frac{5}{9}, \quad P(B) = \frac{6}{11}
\]

and

\[
P(A \cap B) = P(A) \times P(B)
\]
The probability that the target would be hit is given by

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

\[ = \frac{5}{9} + \frac{6}{11} - \frac{10}{33} \]

\[ = \frac{79}{99} \]

Alternately \( P(A \cup B) = 1 - P(A \cup B)' \)

\[ = 1 - P(A' \cap B') \quad \text{(by De-Morgan's Law)} \]

\[ = 1 - P(A') + P(B') \]

\[ = 1 - [1 - P(A)] \times [1 - P(B)] \quad \text{(by 13.30)} \]

\[ = 1 - \left(1 - \frac{5}{9}\right) \times \left(1 - \frac{6}{11}\right) \]

\[ = 1 - \frac{4}{9} \times \frac{5}{11} \]

\[ = \frac{79}{99} \]

**Example 16.16:** A pair of dice is thrown together and the sum of points of the two dice is noted to be 10. What is the probability that one of the two dice has shown the point 4?

**Solution:** Let A denote the event of getting 4 points on one of the two dice and B denote the event of getting a total of 10 points on the two dice. Then we have

\[ P(A) = \frac{1}{2} \times \frac{6}{6} = \frac{1}{12} \]

and \( P(A \cap B) = \frac{2}{36} \)

[Since a total of 10 points may result in (4, 6) or (5, 5) or (6, 4) and two of these combinations contain 4]

Thus \( P(B/A) = \frac{P(A \cap B)}{P(A)} \)
Alternately The sample space for getting a total of 10 points when two dice are thrown simultaneously is given by
\[ S = \{(4, 6), (5, 5), (6, 4)\} \]
Out of these 3 cases, we get 4 in 2 cases. Thus by the definition of probability, we have
\[ P(B/A) = \frac{2}{3} \]

Example 16.17: In a group of 20 males and 15 females, 12 males and 8 females are service holders. What is the probability that a person selected at random from the group is a service holder given that the selected person is a male?

Solution: Let S and M stand for service holder and male respectively. We are to evaluate \( P(S/M) \).

We note that \( S \cap M \) represents the event of both service holder and male.

Thus \( P(S/M) = \frac{P(S \cap M)}{P(M)} \)
\[ = \frac{12}{35} \div \frac{20}{35} \]
\[ = \frac{12}{35} \times \frac{35}{20} = 0.60 \]

Example 16.18: In connection with a random experiment, it is found that
\[ P(A) = \frac{2}{3}, P(B) = \frac{3}{5} \text{ and } P(A \cap B) = \frac{5}{6} \]
Evaluate the following probabilities:
(i) \( P(A/B) \) (ii) \( P(B/A) \) (iii) \( P(A'/ B) \) (iv) \( P(A/ B') \) (v) \( P(A'/ B') \)

Solution: \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \)
\[ \Rightarrow \frac{5}{6} = \frac{2}{3} + \frac{3}{5} - P(A \cap B) \]
\[ \Rightarrow P(A \cap B) = \frac{2}{3} + \frac{3}{5} - \frac{5}{6} \]
\[ = \frac{13}{30} \]
Hence (i) \( P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{13/30}{3/5} = \frac{13}{18} \)

(ii) \( P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{13/30}{2/3} = \frac{13}{20} \)

(iii) \( P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{3}{5} - \frac{13}{30} = \frac{5}{5} = \frac{18}{18} \)

(iv) \( (A/B') = \frac{P(A \cap B')}{P(B')} = \frac{P(B) - P(A \cap B)}{1 - P(B)} = \frac{7}{12} \)

(v) \( P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - 5/6}{1 - 3/5} = \frac{5}{12} \)

Example 16.19: The odds in favour of an event is 2 : 3 and the odds against another event is 3 : 7. Find the probability that only one of the two events occurs.

Solution: We denote the two events by A and B respectively. Then by (16.5) and (16.6), we have

\[
P(A) = \frac{2}{2+3} = \frac{2}{5}
\]
and \( P(B) = \frac{7}{7+3} \cdot \frac{7}{10} \)

As A and B are independent, \( P(A \cap B) = P(A) \times P(B) \)

\[
= \frac{2}{5} \times \frac{7}{10} = \frac{7}{25}
\]

Probability that either only A occurs or only B occurs

\[= P(A - B) + P(B - A)\]

\[= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]\]

\[= P(A) + P(B) - 2 P(A \cap B)\]

\[= \frac{2}{5} + \frac{7}{10} - 2 \times \frac{7}{25}\]

\[= \frac{20 + 35 - 28}{50}\]

\[= \frac{27}{50}\]

**Example 16.20**

There are three boxes with the following compositions:

<table>
<thead>
<tr>
<th>Colour</th>
<th>Blue</th>
<th>Red</th>
<th>White</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5</td>
<td>8</td>
<td>10</td>
<td>23</td>
</tr>
<tr>
<td>II</td>
<td>4</td>
<td>9</td>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>III</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>16</td>
</tr>
</tbody>
</table>

Two balls are drawn from each box. What is the probability that they would be of the same colour?

**Solution:** Either the balls would be Blue or Red or White. Denoting Blue, Red and White balls by B, R and W respectively and the box by lower suffix, the required probability is

\[= P(B_1 \cap B_2 \cap B_3) + P(R_1 \cap R_2 \cap R_3) + P(W_1 \cap W_2 \cap W_3)\]

\[= P(B_1) \times P(B_2) \times P(B_3) + P(R_1) \times P(R_2) \times P(R_3) + P(W_1) \times P(W_2) \times P(W_3)\]

\[= \frac{5}{23} \times \frac{4}{21} \times \frac{3}{16} + \frac{8}{23} \times \frac{9}{21} \times \frac{6}{16} + \frac{10}{23} \times \frac{8}{21} \times \frac{7}{16}\]

\[= \frac{60 + 432 + 560}{7728}\]

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Example 16.21: Mr. Roy is selected for three separate posts. For the first post, there are three candidates, for the second, there are five candidates and for the third, there are 10 candidates. What is the probability that Mr. Roy would be selected?

Solution: Denoting the three posts by A, B and C respectively, we have

\[ P(A) = \frac{1}{3}, \ P(B) = \frac{1}{5} \text{ and } P(C) = \frac{1}{10} \]

The probability that Mr. Roy would be selected (i.e. selected for at least one post).

\[ = P(A \cup B \cup C) \]
\[ = 1 - P[(A \cup B \cup C)'] \]
\[ = 1 - P(A') \times P(B') \times P(C') \] (by De-Morgan’s Law)
\[ = 1 - \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{10}\right) \]

Example 16.22: The independent probabilities that the three sections of a costing department will encounter a computer error are 0.2, 0.3 and 0.1 per week respectively what is the probability that there would be

(i) at least one computer error per week?

(ii) one and only one computer error per week?

Solution: Denoting the three sections by A, B and C respectively, the probabilities of encountering a computer error by these three sections are given by \( P(A) = 0.20, \ P(B) = 0.30 \) and \( P(C) = 0.10 \)

(i) Probability that there would be at least one computer error per week.

\[ = 1 - \text{Probability of having no computer error in any at the three sections.} \]
\[ = 1 - P(A' \cap B' \cap C') \]
\[ = 1 - P(A') \times P(B') \times P(C') \] [Since A, B and C are independent]
\[ = 1 - (1 - 0.20) \times (1 - 0.30) \times (1 - 0.10) \]
\[ = 0.50 \]

(ii) Probability of having one and only one computer error per week

\[ = P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C) \]
\[ = P(A) \times P(B') \times P(C') + P(A') \times P(B) \times P(C') + P(A' \cap B' \cap C) \]
\[ = 0.20 \times 0.70 \times 0.90 + 0.80 \times 0.30 \times 0.90 + 0.80 \times 0.70 \times 0.10 \]
\[ = 0.40 \]
Example 16.23: A lot of 10 electronic components is known to include 3 defective parts. If a sample of 4 components is selected at random from the lot, what is the probability that this sample does not contain more than one detectives?

Solution: Denoting detective component and non-defective components by D and D’ respectively, we have the following situation:

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>D’</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot</td>
<td>3</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Sample (1)</td>
<td>0</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>(2)</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Thus the required probability is given by
\[
\begin{align*}
&= \binom{3}{0} \times \binom{7}{4} \times \binom{3}{1} \times \binom{7}{3} \\
&= \frac{1 \times 35 \times 3 \times 35}{210} \\
&= \frac{2}{3}
\end{align*}
\]

Example 16.24: There are two urns containing 5 red and 6 white balls and 3 red and 7 white balls respectively. If two balls are drawn from the first urn without replacement and transferred to the second urn and then a draw of another two balls is made from it, what is the probability that both the balls drawn are red?

Solution: Since two balls are transferred from the first urn containing 5 red and 6 white balls to the second urn containing 3 red and 7 white balls, we are to consider the following cases:

Case A: Both the balls transferred are red. In this case, the second urn contains 5 red and 7 white balls.

Case B: The two balls transferred are of different colours. Then the second urn contains 4 red and 8 white balls.

Case C: Both the balls transferred are white. Now the second urn contains 3 red and 7 white balls.

The required probability is given by
\[
P(R \cap A) + P(R \cap B) + P(R \cap C) \\
= P(R/A) \times P(A) + P(R/B) \times P(B) + P(R/C) \times P(C) \\
= \frac{5 \binom{2}{5} \times \binom{7}{2}}{12 \binom{2}{12} \times \binom{12}{5}} + \frac{4 \binom{6}{4} \times \binom{8}{2}}{15 \binom{2}{15} \times \binom{15}{6}} + \frac{3 \binom{6}{3} \times \binom{6}{6} \times \binom{7}{2}}{10 \binom{2}{10} \times \binom{10}{5}} \\
= \frac{10 \times 10 + 6 \times 30 + 3 \times 15}{66 \times 55 + 66 \times 55 + 66 \times 55} \\
= \frac{100 + 180 + 45}{3612 + 3612 + 3612} \\
= \frac{325}{10836} \\
= \frac{5}{17}
\]
Example 16.25: If 8 balls are distributed at random among three boxes, what is the probability that the first box would contain 3 balls?

Solution: The first ball can be distributed to the 1st box or 2nd box or 3rd box i.e. it can be distributed in 3 ways. Similarly, the second ball also can be distributed in 3 ways. Thus the first two balls can be distributed in 3^2 ways. Proceeding in this way, we find that 8 balls can be distributed to 3 boxes in 3^8 ways which is the total number of elementary events.

Let A be the event that the first box contains 3 balls which implies that the remaining 5 both must go to the remaining 2 boxes which, as we have already discussed, can be done in 2^5 ways. Since 3 balls out of 8 balls can be selected in \( \binom{8}{3} \) ways, the event can occur in \( \binom{8}{3} \times 2^5 \) ways, thus we have

\[
P(A) = \frac{\binom{8}{3} \times 2^5}{3^8}
\]

\[
= \frac{56 \times 32}{6561}
\]

\[
= \frac{1792}{6561}
\]

Example 16.26: There are 3 boxes with the following composition:

- Box I: 7 Red + 5 White + 4 Blue balls
- Box II: 5 Red + 6 White + 3 Blue balls
- Box III: 4 Red + 3 White + 2 Blue balls

One of the boxes is selected at random and a ball is drawn from it. What is the probability that the drawn ball is red?

Solution: Let A denote the event that the drawn ball is blue. Since any of the 3 boxes may be drawn, we have \( P(B) = P(B_{II}) = P(B_{III}) = \frac{1}{3} \)

Also \( P(R_{I}/B_{II}) = \) probability of drawing a red ball from the first box

\[
= \frac{7}{16}
\]

\( P(R_{II} / B_{II}) = \frac{5}{14} \) and \( P(R_{III} / B_{III}) = \frac{4}{9} \)

Thus we have

\[
P(A) = P(R_{I} \cap B_{I}) + P(R_{II} \cap B_{II}) + P(R_{III} \cap B_{III})
\]
\[ P(R_1 / B_1) \times P(B_1) + P(R_2 / B_2) \times P(B_2) + P(R_3 / B_3) \times P(B_3) \]
\[ = \frac{7}{16} + \frac{5}{14} + \frac{4}{9} = \frac{1249}{3024} \]

16.9 Random-Variable-Probability Distribution

A random variable or stochastic variable \( R \) is considered as a collection of outcomes of a random experiment. Let \( S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\} \) be the sample space, and we find that \( X = 0 \) if the sample point is \( TTT \), \( X = 1 \) if the sample point is \( HTT, THT, \) or \( TTH \), \( X = 2 \) if the sample point is \( HHT, HTH, \) or \( THH \), and \( X = 3 \) if the sample point is \( HHH \).

\( X_1, X_2, X_3, \ldots, X_n \) are random variables, and \( P_1, P_2, P_3, \ldots, P_n \) are the corresponding probabilities. Then the probability distribution of the random variable \( X \) is given by

\[ p_i \geq 0 \quad \text{for every } i \] \hspace{1cm} (16.33)

and \( \sum p_i = 1 \) \hspace{1cm} (over all \( i \)) \hspace{1cm} (16.34)

then the probability distribution of the random variable \( X \) is given by
Probability Distribution of X

<table>
<thead>
<tr>
<th>X</th>
<th>X_1</th>
<th>X_2</th>
<th>X_3</th>
<th>......X_n</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>P_1</td>
<td>P_2</td>
<td>P_3</td>
<td>...... P_n</td>
<td>1</td>
</tr>
</tbody>
</table>

For example, if an unbiased coin is tossed three times and if X denotes the number of heads then, as we have already discussed, X is a random variable and its probability distribution is given by

Probability Distribution of Head when a Coin is Tossed Thrice

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1/8</td>
<td>3/8</td>
<td>3/8</td>
<td>1/8</td>
<td>1</td>
</tr>
</tbody>
</table>

(i) \( f(X) \geq 0 \) for every \( X \)  

and (ii) \( \sum f(X) = 1 \)

Where \( f(X) \) is given by

\( f(X) = P(X = X) \)

(i) \( f(x) \geq 0 \) for \( x \in [\alpha, \beta] \) 

(ii) \( \int_{\alpha}^{\beta} f(x) \, dx = 1 \)
16.10 एक दैविक क्षण का प्रत्याशित मूल्य (Expected Value of a Random Variable)

एक दैविक क्षण का प्रत्याशित मूल्य या गणनीय प्रत्याशा या संभावना को परिभाषित किया जा सकता है दैविक तथा तत्समक्ष संभावनाओं द्वारा लिये गये विभिन्न मूल्यों के गुणनफलों के योग के रूप में। अतः यदि एक दैविक चयन x के लेसर है n मूल्य x₁, x₂, x₃, . . . . . . xₙ तत्समक्ष संभावनाओं P₁, P₂, P₃, . . . . . , Pₙ के साथ, तो ते P₁ संतुष्ट करता है (13.33) तथा (13.34) को, तो x का प्रत्याशित मूल्य दिया जायेगा—

\[ \mu = E(x) = \sum p_i x_i \] 

\[ \text{Expected value of } x^2 \text{ in given by} \]

\[ E(x^2) = \sum p_i x_i^2 \] 

In particular expected value of a monotonic function \( g(x) \) is given by

\[ E[g(x)] = \sum p_i g(x_i) \] 

Variance of x, to be denoted by \( \sigma^2 \) is given by

\[ V(x) = \sigma^2 = E(x - \mu)^2 \]

\[ = E(x^2) - \mu^2 \] 

\[ \text{The positive square root of variance is known as standard deviation and is denoted by } \sigma. \]

If \( y = a + bx \), for two random variables x and y and for a pair of constants \( a \) and \( b \), then the mean i.e. expected value of \( y \) is given by

\[ \mu_y = a + b \mu_x \] 

and the standard deviation of \( y \) is

\[ \sigma_y = |b| \times \sigma_x \] 

When \( x \) is a discrete random variable with probability mass function \( f(x) \), then its expected value is given by

\[ \mu = \sum_{x} x f(x) \] 

and its variance is

\[ \sigma^2 = E(x^2) - \mu^2 \]

\[ \text{Where } E(x^2) = \sum_{x} x^2 f(x) \] 

For a continuous random variable \( x \) defined in \([ , ]\), its expected value (i.e. mean) and variance are given by
E (x) = \int_{a}^{b} x f(x) dx \quad \ldots \ldots (16.49)

and \( \sigma^2 = E(x^2) - \mu^2 \)

where

E (x^2) = \int_{a}^{b} x^2 f(x) dx \quad \ldots \ldots (16.50)

\( \text{Example 16.27:} \) An unbiased coin is tossed three times. Find the expected value of the number of heads and also its standard deviation.

\( \text{Solution:} \) If \( x \) denotes the number of heads when an unbiased coin is tossed three times, then the probability distribution of \( x \) is given by

<table>
<thead>
<tr>
<th>( X ) :</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P ) :</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{3}{8} )</td>
<td>( \frac{3}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
</tbody>
</table>

The expected value of \( x \) is given by

\[ \mu = E(x) = \sum p_i x_i = \frac{1}{8} \times 0 + \frac{3}{8} \times 1 + \frac{3}{8} \times 2 + \frac{1}{8} \times 3 = 0 + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1.50 \]
Also \( E(x^2) = \sum p_i x_i^2 \)

\[
\begin{align*}
&= \frac{1}{8} \times 0^2 + \frac{3}{8} \times 1^2 + \frac{3}{8} \times 2^2 + \frac{1}{8} \times 3^2 \\
&= \frac{0 + 3 + 12 + 9}{8} = 3
\end{align*}
\]

\[
\sigma^2 = E(x^2) - \mu^2 = 3 - (1.50)^2 = 0.75
\]

\[
\therefore \text{SD} = \sigma = 0.87
\]

**Example 16.28:** A random variable has the following probability distribution:

<table>
<thead>
<tr>
<th>( X )</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>0.15</td>
<td>0.20</td>
<td>0.40</td>
<td>0.15</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Find \( E[x - E(x)]^2 \). Also obtain \( \text{var}(3x - 4) \)

**Solution:** The expected value of \( x \) is given by

\[
E(x) = \sum p_i x_i
\]

\[
= 0.15 \times 4 + 0.20 \times 5 + 0.40 \times 7 + 0.15 \times 8 + 0.10 \times 10
\]

\[
= 6.60
\]

Also, \( E[x - E(x)]^2 = \sum \mu_i^2 P_i \) where \( \mu_i = x_i - E(x) \)

Let \( y = 3x - 4 = (-4) + (3)x \). Then variance of \( y = \text{var} y = b^2 \times \sigma_i^2 = 9 \times \mu_i^2 \) (From 13.46)

**Table 16.1**

<table>
<thead>
<tr>
<th>( x_i )</th>
<th>( p_i )</th>
<th>( \mu_i = x_i - E(x) )</th>
<th>( \mu_i^2 )</th>
<th>( \mu_i^2 P_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.15</td>
<td>-2.60</td>
<td>6.76</td>
<td>1.014</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
<td>-1.60</td>
<td>2.56</td>
<td>0.512</td>
</tr>
<tr>
<td>7</td>
<td>0.40</td>
<td>0.40</td>
<td>0.16</td>
<td>0.064</td>
</tr>
<tr>
<td>8</td>
<td>0.15</td>
<td>1.40</td>
<td>1.96</td>
<td>0.294</td>
</tr>
<tr>
<td>10</td>
<td>0.10</td>
<td>3.40</td>
<td>11.56</td>
<td>1.156</td>
</tr>
<tr>
<td>Total</td>
<td>1.00</td>
<td></td>
<td>3.040</td>
<td></td>
</tr>
</tbody>
</table>
Thus $E [x - E(x)]^2 = 3.04$

As $\mu^2 = 3.04$, $v(y) = 9 \times 3.04 = 27.36$

**Example 16.29:** In a business venture, a man can make a profit of ₹ 50,000 or incur a loss of ₹ 20,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.75 and 0.25 respectively. What is his expected profit?

**Solution:** If the profit is denoted by x, then we have the following probability distribution of x:

<table>
<thead>
<tr>
<th>X</th>
<th>₹ 50,000</th>
<th>₹ –20,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.75</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Thus his expected profit

$$E(x) = p_1 x_1 + p_2 x_2$$

$$= 0.75 \times ₹ 50,000 + 0.25 \times (₹ –20,000)$$

$$= ₹ 32,500$$

**Example 16.30:** A box contains 12 electric lamps of which 5 are defectives. A man selects three lamps at random. What is the expected number of defective lamps in his selection?

**Solution:** Let $x$ denote the number of defective lamps $x$ can assume the values 0, 1, 2 and 3.

$$P(x = 0) = \text{Prob. of having 0 defective out of 5 defectives and 3 non defective out of 7 non defectives}$$

$$= \frac{\binom{5}{0} \times \binom{7}{3} \times \binom{3}{3}}{\binom{12}{3}} = \frac{35}{220}$$

Similarly

$$P(x = 1) = \frac{\binom{5}{1} \times \binom{7}{2} \times \binom{3}{3}}{\binom{12}{3}} = \frac{105}{220}$$

$$P(x = 2) = \frac{\binom{5}{2} \times \binom{7}{1} \times \binom{3}{3}}{\binom{12}{3}} = \frac{70}{220}$$

and

$$P(x = 3) = \frac{\binom{5}{3} \times \binom{7}{0} \times \binom{3}{3}}{\binom{12}{3}} = \frac{10}{220}$$

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$\frac{35}{220}$</td>
<td>$\frac{105}{220}$</td>
<td>$\frac{70}{220}$</td>
<td>$\frac{10}{220}$</td>
</tr>
</tbody>
</table>

Thus the expected number of defectives is given by
Example 16.31: Moidul draws 2 balls from a bag containing 3 white and 5 Red balls. He gets ₹ 500 if he draws a white ball and ₹ 200 if he draws a red ball. What is his expectation? If he is asked to pay ₹ 400 for participating in the game, would he consider it a fair game and participate?

Solution: We denote the amount by x. Then x assumes the value 2 x ₹ 500 i.e. ₹ 1000 if 2 white balls are drawn, the value ₹ 500 + ₹ 200 i.e. ₹ 700 if 1 white and 1 red balls are drawn and the value 2 x ₹ 200 i.e. ₹ 400 if 2 red balls are drawn. The respective probabilities are given by

\[
P(WW) = \frac{\binom{3}{2}}{\binom{8}{2}} = \frac{3}{28}
\]

\[
P(WR) = \frac{\binom{3}{1} \times \binom{5}{1}}{\binom{8}{2}} = \frac{15}{28}
\]

and \[
P(RR) = \frac{\binom{5}{2}}{\binom{8}{2}} = \frac{10}{28}
\]

**Probability Distribution of x**

<table>
<thead>
<tr>
<th>X</th>
<th>₹ 1000</th>
<th>₹ 700</th>
<th>₹ 400</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>\frac{3}{28}</td>
<td>\frac{15}{28}</td>
<td>\frac{10}{28}</td>
</tr>
</tbody>
</table>

Hence \( E(x) = \frac{3}{28} \times ₹ 1000 + \frac{15}{28} \times ₹ 700 + \frac{10}{28} \times ₹ 400 \)

\[= \frac{3000 + 10500 + 4000}{28} = ₹ 625\]

Example 16.32: A number is selected at random from a set containing the first 100 natural numbers and another number is selected at random from another set containing the first 200 natural numbers. What is the expected value of the product?

Solution: We denote the number selected from the first set by x and the number selected from the second set by y. Since the selections are independent of each other, the expected value of the product is given by

\[E(xy) = E(x) \times E(y) \quad \text{........... (1)}\]

Now x can assume any value between 1 to 100 with the same probability 1/100 and as such
the probability distribution of \( x \) is given by

\[
\begin{array}{ccc}
X & 1 & 2 & 3 & \ldots & 100 \\
\text{P} & \frac{1}{100} & \frac{1}{100} & \frac{1}{100} & \ldots & \frac{1}{100} \\
\end{array}
\]

Thus \( E(x) = \frac{1}{100} \times 1 + \frac{1}{100} \times 2 + \frac{1}{100} \times 3 + \ldots + \frac{1}{100} \times 100 \)

\[
= \frac{1+2+3+\ldots+100}{100}
\]

\[
= \frac{100 \times 101}{2 \times 100} \quad \text{[Since } 1+2+\ldots+n = \frac{n(n+1)}{2} \text{]}
\]

\[
= \frac{101}{2}
\]

Similarly, \( E(y) = \frac{201}{2} \)

\[
\therefore E(xy) = \frac{101}{2} \times \frac{201}{2} \quad \text{[From (1)]}
\]

\[
= \frac{20301}{4}
\]

\[
= 5075.25
\]

**Example 16.33:** A dice is thrown repeatedly till a 'six' appears. Write down the sample space. Also find the expected number of throws.

**Solution:** Let \( p \) denote the probability of getting a six and \( q = 1 - p \), the probability of not getting a six. If the dice is unbiased then

\[
p = \frac{1}{6} \quad \text{and} \quad q = \frac{5}{6}
\]

यदि एकदम पहले ही फैक्टर्स में 6 आ जाता है तो प्रयोग खत्म हो जाता है तथा यदि छ: पाने की संभावना, जैसा कि हमने पहले ही देखा है \( p \) होगी। लेकिन यदि पहलर छोए छ: नहीं आ जाता तो पास की संभावना यदि छ: आ जाता है तो प्रयोग चलता जायेंगा। एक गैर-छ: से ही फैले छ: पाने की संभावना है \( q \)। यदि दूसरे छोरे से 6 नहीं आता है हम सीमारे छोरे करेंगे तथा यदि छ: हम सींसरे छोरे में 6 आ जाता है तो प्रयोग समाप्त हो जायेंगा। तथा 6 पाने की संभावना में संभावना है \( q^2p \) प्रयोग चलता जायेंगा।

\[
S = \{ p, qp, q^2p, q^3p, \ldots \}
\]
If x denotes the number of throws necessary to produce a six, then x is a random variable with the following probability distribution:

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>……….</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>p</td>
<td>qp</td>
<td>q2p</td>
<td>q3p</td>
<td>……….</td>
</tr>
</tbody>
</table>

Thus \( E(x) = p \times 1 + qp \times 2 + q^2p \times 3 + q^3p \times 4 + \ldots \ldots \ldots \)

\[ E(x) = p(1 + 2q + 3q^2 + 4q^3 + \ldots \ldots \ldots) \]

\[ E(x) = p \frac{1 - q^{-2}}{p} \]

\[ E(x) = \frac{1}{q} \]

In case of an unbiased dice, \( p = \frac{1}{6} \) and \( E(x) = 6 \)

**Example 16.34:** A random variable x has the following probability distribution:

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>0</td>
<td>2k</td>
<td>3k</td>
<td>k</td>
<td>2k</td>
<td>k2</td>
<td>7k^2</td>
<td>2k^2+k</td>
</tr>
</tbody>
</table>

Find (i) the value of k
(ii) \( P(x < 3) \)
(iii) \( P(x \geq 4) \)
(iv) \( P(2 < x \geq 5) \)

**Solution:** By virtue of (13.36), we have

\[ \sum P(x) = 1 \]

\[ \Rightarrow 0 + 2k + 3k + k + 2k + k^2 + 7k^2 + 2k^2 + k = 1 \]

\[ \Rightarrow 10k^2 + 9k - 1 = 0 \]

\[ \Rightarrow (k + 1)(10k - 1) = 0 \]

\[ \Rightarrow k = 1/10 \] (as \( k \neq -1 \) by virtue of (16.36))

(i) Thus the value of k is 0.10
(ii) \( P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2) \)

\[ = 0 + 2k + 3k \]

\[ = 5k \]

\[ = 0.50 \] (as \( k = 0.10 \))

(iii) \( P(x \geq 4) = P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7) \)
= 2k + k² + 7k² + (2k² + k)
= 10k² + 3k
= 10 x (0.10)² + 3 x 0.10
= 0.40

(iv) \( P(x < x \geq 5) = P(x = 3) + P(x = 4) + P(x = 5) \)
= k + 2k² + k²
= k² + 3k
= (0.10)² + 3 x 0.10
= 0.31

**सारांश (Summary)**

- **प्रयोग (experiment)**: एक प्रयोग को परिभाषित किया जा सकता है एक निष्पत्ति के रूप में जो कठिन परिणाम उत्पन्न कर।
- **दैव प्रयोग (Random Experiment)**: एक प्रयोग को दैव के रूप में परिभाषित किया जाता है यदि उसमें परिणाम केवल अवसर पर निर्भर करते हों।
- **घटनाएँ (Events)**: एक दैव प्रयोग के परिणाम घटनाएं के रूप में जाने जाते हैं क्योंकि केवल घटनाएं परिणाम का संयोजन हो सकता है। घटनाएं दो प्रकार की होती हैं।
  (i) साधारण या प्रामाणिक।
  (ii) निहित या संकीर्ण।
- **परस्पर अवलम्बी घटनाएँ (Mutually Exclusive Events or Incompatible Events)**: घटनाएं का केवल समुच्चय \( A_1, A_2, A_3, \ldots \) परस्पर अवलम्बी के रूप में जाना जाता है यदि उनमें से एक से अवलम्बक साथ साथ उपन्न नहीं हो सकता।
- **निष्पादकीय घटनाएँ (Exhaustive Events)**: घटनाएं \( A_1, A_2, A_3, \ldots \) एक exhaustive set का निर्माण करती हैं जानी जाती हैं। यदि इन घटनाओं में से एक असंभवत: उपन्न होनी चाहिए।
- **समान रूप से सम्भव घटनाएँ (Equally likely events या mutually Symmetric Events या Equi-Probable Events)**: एक दैव प्रयोग का घटनाओं को समान तौर पर संयोजित माना जाता है जब सभी आवश्यक साथ साथ ध्यान में रखे जाते हैं, किसी घटना के अवलम्बक बार बार उपन्न होने की आशा न हो घटनाओं के सेट की अन्य घटनाओं की तुलना में।
- **Event A के होने का प्रामाणित करना** का परिभाषित किया जाता है घटनाओं की कुल संख्या के प्रति \( A \) के प्रति अनुकूल घटनाओं की संख्या के अनुपात के रूप में। इसको \( P(A) \) द्वारा व्यक्त हम पाते हैं के नतीजे, \[ P(A) = \frac{n_1}{n} = \frac{\text{No. of equally likely events favourable to } A}{\text{Total no. of equally likely events}} \]
(a) \(0 \leq P(A) \leq 1\)
When \(P(A) = 0\), \(A\) is known to be an impossible event and when \(P(A) = 1\), \(A\) is known to be a sure event.

(b) Event \(A\) के न घटना का \(A'\) द्वारा \(A\)' या \(-A\) द्वारा व्यक्त किया जाता है तथा इसको \(A\) को पूरक घटना माना जाता है। घटना \(A\) अन्य पूरक \(A'\) के साथ परस्पर अपरिवर्तनीय तथा Exhaustive घटनाओं के एक सेट का निर्माण करते हैं।
i.e. \(P(A) + P(A') = 1\)
\(\Rightarrow P(A') = 1 - P(A)\)
\[
1 - \frac{m_A}{m} = \frac{m - m_A}{m}
\]

(c) अनुकूल घटनाओं के प्रति प्रतिकूल घटनाओं की संख्या का अनुपात माना जाता है Odds in favour of the event \(A\) तथा दूसरी inverse ratio को कहा जाता है Odds against the event \(A\).
Arithmetically odds in favour of \(A\) = \(m_A : (m - m_A)\)
and odds against \(A\) = \((m - m_A) : m_A\)

(d) किन्हीं दो परस्पर अपरिवर्तनीय घटनाओं \(A\) तथा \(B\) द्वारा यह प्राप्तिकर्ता चूके \(A\) या \(B\) परिवर्तित होगी।
\(A\) तथा \(B\) को व्यक्तिगत प्राप्तिकर्ताओं के योग द्वारा दी जाती है।
\(P(A \cup B)\)
or \(P(A + B) = P(A) + P(B)\)

(e) For any \(K(\geq 2)\) mutually exclusive events \(A_1, A_2, A_3, \ldots, A_K\) the probability that at least one of them occurs is given by the sum of the individual probabilities of the \(K\) events.
i.e. \(P(A_1 \cup A_2 \cup \ldots \cup A_K) = P(A_1) + P(A_2) + \ldots. P(A_K)\)

(f) किन्हीं दो घटनाओं \(A\) तथा \(B\) के लिए यह प्राप्तिकर्ता चूके \(A\) तथा \(B\) साथ साथ उत्पन्न होती है।
\(A\) तथा \(B\) को व्यक्तिगत प्राप्तिकर्ताओं के योग द्वारा दी जाती है घटनाओं \(A\) तथा \(B\) घटनाओं का साथ साथ आवृति (simultaneous occurrence) का प्राप्तिकर्ता को।
i.e. \(P(A \cup B) = P(A) + P(B) - P(A \cap B)\)

(g) किन्हीं तीन घटनाओं \(A, B\) तथा \(C\) लिए यह प्राप्तिकर्ता कि कम से कम एक घटना उत्पन्न होती है को ........... द्वारा दिया जाता है
\(P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)\)

(h) किन्हीं दो घटनाओं \(A\) तथा \(B\) के लिए यह प्राप्तिकर्ता \(A\) तथा \(B\) साथ साथ उत्पन्न होती है को दिया जाता है कि \(A\) शर्तीय प्राप्तिकर्ता तथा \(B\) को सरल प्राप्तिकर्ता को गुणांक द्वारा यह मानते हुए कि \(A\) पहले उत्पन्न हो चुकी है।
i.e. \( P(A \cap B) = P(A) \times P(B/A) \) Provided \( P(A) > 0 \)

(i) संयुक्त प्रायिकता (Compound Probability या Joint Probability)

\[
P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}
\]

(j) किन्हीं तीन घटनाओं \( A, B \) तथा \( C \) के लिए यह प्रायिकता के संयुक्त तौर पर उत्पन्न होती है को दिया जाता है।

\[ P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/(A \cap B)) \] Provided \( P(A \cap B) > 0 \)

(k) \( P(A/B) = \frac{P(A \cap B)}{P(B)} \)

\[
P(B/A) = \frac{P(A \cap B)}{P(A)}
\]

\[
P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}
\]

(l) \( P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(A \cap B)}{1 - P(B)} \)

(m) \( P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cap B')'}{P(B')} \)

\[
= \frac{P(A \cup B')}{P(B')} \text{ [by De-Morgan’s Law } A' \cap B' = (A \cup B)’]\]

\[
= \frac{1 - P(A \cup B)}{1 - P(B)}
\]

- एक रैंडम वरियल (Random Variable या stochastic Variable) : एक function है एक दैव प्रयोग के साथ जुड़ी Sample space पर परिभाषित यह मानते हैं कि \( R \) से कोई मूल्य तथा दैव प्रयोग के प्रत्येक सौपिल Poin्त के प्रति एक वास्तविक संख्या देते हैं।
- प्रवृत्ति मूल्य या Mathematic Expectation of a random variable को परिभाषित किया जा सकता है दैव तथा तत्त्वमाल समानांतरों द्वारा लिये गये मूल्यों के उल्लम्ब के और के रूप में।
When \( x \) is a discrete random variable with probability mass function \( f(x) \),
then its expected value is given by
\[
\mu = \sum_x x f(x)
\]
and its variance is
\[
\sigma^2 = E(x^2) - \mu^2
\]
Where \( E(x^2) = \sum_x x^2 f(x) \)

For a continuous random variable \( x \) defined in \( [-\infty, \infty] \), its expected value (i.e. mean) and variance are given by
\[
E(x) = \int_{-\infty}^{\infty} x f(x) dx
\]
and \( \sigma^2 = E(x^2) - \mu^2 \)
where \( E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx \)

Properties of Expected Values

(i) Expectation of a constant \( k \) is \( k \)
   i.e. \( E(k) = k \) for any constant \( k \).

(ii) Expectation of sum of two random variables is the sum of their expectations.
   i.e. \( E(x + y) = E(x) + E(y) \) for any two random variables \( x \) and \( y \).

(iii) Expectation of the product of a constant and a random variable is the product
   of the constant and the expectation of the random variable.
   i.e. \( E(kx) = kE(x) \) for any constant \( k \).

(iv) दो दैवचरों के गुणनफल को प्रत्यक्षा – दो दैवचरों की प्रत्यक्षा का गुणनफल होता है बसाते है कि दोनों
    चार स्वतंत्र हो।
    i.e. \( E(xy) = E(x) \times E(y) \)
    जब भी \( x \) तथा \( y \) स्वतंत्र हो।