15.2.1 DEFINITION OF DISPERSION

The second important characteristic of a distribution is given by dispersion. Two distributions may be identical in respect of its first important characteristic i.e. central tendency and yet they may differ on account of scatterness. The following figure shows a number of distributions having identical measure of central tendency and yet varying measure of scatterness. Obviously, distribution is having the maximum amount of dispersion.

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Dispersion for a given set of observations may be defined as the amount of deviation of the observations, usually, from an appropriate measure of central tendency. Measures of dispersion may be broadly classified into

1. **Absolute measures of dispersion.**

   Absolute measures of dispersion are classified into
   (i) Range
   (ii) Mean Deviation
   (iii) Standard Deviation
   (iv) Quartile Deviation

   Likewise, we have the following relative measures of dispersion:
   (i) Coefficient of Range.
   (ii) Coefficient of Mean Deviation
   (iii) Coefficient of Variation
   (iv) Coefficient of Quartile Deviation.

We may note the following points of distinction between the absolute and relative measures of dispersion:

I Absolute measures are dependent on the unit of the variable under consideration whereas the relative measures of dispersion are unit free.

II For comparing two or more distributions, relative measures and not absolute measures of dispersion are considered.

III Compared to absolute measures of dispersion, relative measures of dispersion are difficult to compute and comprehend.

**Characteristics for an ideal measure of dispersion**

As discussed in section 15.2.1 an ideal measure of dispersion should be properly defined, easy to comprehend, simple to compute, based on all the observations, unaffected by sampling fluctuations and amenable to some desirable mathematical treatment.
15.2.2 RANGE

For a given set of observations, range may be defined as the difference between the largest and smallest of observations. Thus if \( L \) and \( S \) denote the largest and smallest observations respectively then we have

\[
\text{Range} = L - S
\]

The corresponding relative measure of dispersion, known as coefficient of range, is given by

\[
\text{Coefficient of range} = \frac{L - S}{L + S} \times 100
\]

For a grouped frequency distribution, range is defined as the difference between the two extreme class boundaries. The corresponding relative measure of dispersion is given by the ratio of the difference between the two extreme class boundaries to the total of these class boundaries, expressed as a percentage.

We may note the following important result in connection with range:

**Result:**

Range remains unaffected due to a change of origin but affected in the same ratio due to a change in scale i.e., if for any two constants \( a \) and \( b \), two variables \( x \) and \( y \) are related by \( y = a + bx \),

Then the range of \( y \) is given by

\[
R_y = |b| \times R_x \quad \text{................................................................. (15.2.1)}
\]

**Example 15.2.1:** Following are the wages of 8 workers expressed in Rupees.
82, 96, 52, 75, 70, 65, 50, 70. Find the range and also its coefficient.

**Solution:** The largest and the smallest wages are \( L = ₹ 96 \) and \( S = ₹ 50 \)
Thus range = \( ₹ 96 - ₹ 50 = ₹ 46 \)

Coefficient of range = \( \frac{96 - 50}{96 + 50} \times 100 \)
= 31.51

**Example 15.2.2:** What is the range and its coefficient for the following distribution of weights?

<table>
<thead>
<tr>
<th>Weights in kgs.</th>
<th>50 – 54</th>
<th>55 – 59</th>
<th>60 – 64</th>
<th>65 – 69</th>
<th>70 – 74</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Students</td>
<td>12</td>
<td>18</td>
<td>23</td>
<td>10</td>
<td>3</td>
</tr>
</tbody>
</table>

**Solution:** The lowest class boundary is 49.50 kgs. and the highest class boundary is 74.50 kgs. Thus we have

Range = 74.50 kgs. – 49.50 kgs.
= 25 kgs.

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Also, coefficient of range = \[ \frac{74.50 - 49.50}{74.50 + 49.50} \times 100 \]
\[ = \frac{25}{124} \times 100 \]
\[ = 20.16 \]

**Example 15.2.3:** If the relationship between \( x \) and \( y \) is given by \( 2x + 3y = 10 \) and the range of \( x \) is \( \mathbf{15} \), what would be the range of \( y \)?

**Solution:** Since \( 2x + 3y = 10 \)

Therefore,
\[ y = \frac{10}{3} - \frac{2}{3} x \]

Applying (11.23), the range of \( y \) is given by
\[ R_y = |b| \times R_x \]
\[ = \frac{2}{3} \times \mathbf{15} \]
\[ = \mathbf{10}. \]

### 15.2.3 MEAN DEVIATION

Since range is based on only two observations, it is not regarded as an ideal measure of dispersion. A better measure of dispersion is provided by mean deviation which, unlike range, is based on all the observations. For a given set of observation, mean deviation is defined as the arithmetic mean of the absolute deviations of the observations from an appropriate measure of central tendency. Hence if a variable \( x \) assumes \( n \) values \( x_1, x_2, x_3, \ldots, x_n \), then the mean deviation of \( x \) about an average \( A \) is given by

\[ \text{MD}_A = \frac{1}{n} \sum |x_i - A| \] \hspace{1cm} \text{(15.2.2)}

For a grouped frequency distribution, mean deviation about \( A \) is given by

\[ \text{MD}_A = \frac{1}{n} \sum |x_i - A| f_i \] \hspace{1cm} \text{(15.2.2)}

Where \( x_i \) and \( f_i \) denote the mid value and frequency of the \( i \)-th class interval and

\[ N = \sum f_i \]

In most cases we take \( A \) as mean or median and accordingly, we get mean deviation about mean or mean deviation about median.

A relative measure of dispersion applying mean deviation is given by
Mean deviation takes its minimum value when the deviations are taken from the median. Also mean deviation remains unchanged due to a change of origin but changes in the same ratio due to a change in scale i.e. if \( y = a + bx \), \( a \) and \( b \) being constants, then \( \text{MD of } y = |b| \times \text{MD of } x \) ………………………(15.2.4)

Example 15.2.4: What is the mean deviation about mean for the following numbers?
5, 8, 10, 10, 12, 9.

Solution:
The mean is given by
\[
\bar{x} = \frac{5 + 8 + 10 + 10 + 12 + 9}{6} = 9
\]

Table 15.2.1

| \( x_i \) | \( |x_i - \bar{x}| \) |
|----------|-----------------|
| 5        | 4               |
| 8        | 1               |
| 10       | 1               |
| 10       | 1               |
| 12       | 3               |
| 9        | 0               |
| **Total** | **10**          |

Thus mean deviation about mean is given by
\[
\frac{\sum| |x_i - \bar{x}|}{n} = \frac{10}{6} = 1.67
\]

Example. 15.2.5: Find mean deviations about median and also the corresponding coefficient for the following profits (‘000 ₹) of a firm during a week.
82, 56, 75, 70, 52, 80, 68.

Solution:
The profits in thousand rupees is denoted by \( x \). Arranging the values of \( x \) in an ascending order, we get
52, 56, 68, 70, 75, 80, 82.
Therefore, $Me = 70$. Thus, Median profit = ₹ 70,000.

Table 15.2.2

| $x_i$ | $|x_i - Me|$ |
|-------|------------|
| 52    | 18         |
| 56    | 14         |
| 68    | 2          |
| 70    | 0          |
| 75    | 5          |
| 80    | 10         |
| 82    | 12         |
| Total | 61         |

Thus mean deviation about median = $\frac{\sum |x_i - \text{Median}|}{n}$

$= (\text{₹}) \frac{61}{7}$
$= \text{₹} 8714.28$

Coefficient of mean deviation = $\frac{\text{MD about median}}{\text{Median}} \times 100$

$= \frac{8714.28}{70000} \times 100$
$= 12.45$

Example 15.2.6: Compute the mean deviation about the arithmetic mean for the following data:

\[ x: \begin{array}{cccccc} 1 & 3 & 5 & 7 & 9 \\ f: \begin{array}{cccccc} 5 & 8 & 9 & 2 & 1 \end{array} \end{array} \]

Also find the coefficient of the mean deviation about the AM.
**Solution:** We are to apply formula (15.1.2) as these data refer to a grouped frequency distribution the AM is given by

\[
\bar{x} = \frac{\sum f_i x_i}{N}
\]

\[
= \frac{5 \times 1 + 8 \times 3 + 9 \times 5 + 2 \times 7 + 1 \times 9}{5 + 8 + 9 + 2 + 1} = 3.88
\]

| Table 15.2.3 |
| Computation of MD about the AM |
| (1) | (2) | (3) | (4) = (2) \times (3) |
| 1 | 5 | 2.88 | 14.40 |
| 3 | 8 | 0.88 | 7.04 |
| 5 | 9 | 1.12 | 10.08 |
| 7 | 2 | 3.12 | 6.24 |
| 9 | 1 | 5.12 | 5.12 |
| Total | 25 | – | 42.88 |

Thus, MD about AM is given by

\[
\frac{\sum f_i |x - \bar{x}|}{N} = \frac{42.88}{25} = 1.72
\]

**Coefficient of MD about its AM**

\[
= \frac{1.72 \times 100}{3.88} = 44.33
\]

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Example 15.2.7: Compute the coefficient of mean deviation about median for the following distribution:

Weight in kgs. : 40-50 50-60 60-70 70-80
No. of persons : 8    12   20    10

Solution: We need to compute the median weight in the first stage

<table>
<thead>
<tr>
<th>Weight in kg (CB)</th>
<th>No. of Persons (Cumulative Frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>8</td>
</tr>
<tr>
<td>60</td>
<td>20</td>
</tr>
<tr>
<td>70</td>
<td>40</td>
</tr>
<tr>
<td>80</td>
<td>50</td>
</tr>
</tbody>
</table>

Hence,

\[
M = l_1 + \left( \frac{N - N_l}{N_u - N_l} \right) \times C
\]

\[
= \left[ 60 + \frac{25 - 20}{40 - 20} \times 10 \right] \text{kg.} = 62.50 \text{kg.}
\]

| weight (kgs.) (1) | mid-value \((x_i)\) kgs. (2) | No. of persons (3) | \(|x_i - \text{Me}| \) (kgs.) (4) | \(f_i|x_i - \text{Me}| \) (kgs.) (5) = (3) \times (4) |
|-------------------|------------------------------|--------------------|-------------------------------|-----------------------------------|
| 40–50             | 45                           | 8                  | 17.50                         | 140                               |
| 50–60             | 55                           | 12                 | 7.50                          | 90                                |
| 60–70             | 65                           | 20                 | 2.50                          | 50                                |
| 70–80             | 75                           | 10                 | 12.50                         | 125                               |
| Total             | –                            | 50                 | –                             | 405                               |

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Mean deviation about median = \[ \frac{\sum f_i|x_i - \text{Median}|}{N} \]

\[ = \frac{405}{50} \text{ kg.} \]

\[ = 8.10 \text{ kg.} \]

Coefficient of mean deviation about median = \[ \frac{\text{Mean deviation about median}}{\text{Median}} \times 100 \]

\[ = \frac{8.10}{62.50} \times 100 \]

\[ = 12.96 \]

**Example 15.2.8**: If \( x \) and \( y \) are related as \( 4x + 3y + 11 = 0 \) and mean deviation of \( x \) is 5.40, what is the mean deviation of \( y \)?

**Solution**: Since \( 4x + 3y + 11 = 0 \)

Therefore, \( y = \left( -\frac{11}{3} \right) + \left( -\frac{4}{3} \right)x \)

Hence MD of \( y = |b| \times \text{MD of } x \)

\[ = \frac{4}{3} \times 5.40 \]

\[ = 7.20 \]

15.2.4 STANDARD DEVIATION

Although mean deviation is an improvement over range so far as a measure of dispersion is concerned, mean deviation is difficult to compute and furthermore, it cannot be treated mathematically. The best measure of dispersion is, usually, standard deviation which does not possess the demerits of range and mean deviation.

Standard deviation for a given set of observations is defined as the root mean square deviation when the deviations are taken from the AM of the observations. If a variable \( x \) assumes \( n \) values \( x_1, x_2, x_3 \ldots \ldots \ldots x_n \) then its standard deviation(s) is given by

\[ s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} \] ..............................(15.2.5)

For a grouped frequency distribution, the standard deviation is given by

\[ s = \sqrt{\frac{\sum f_i(x_i - \bar{x})^2}{N}} \] .............................. (15.2.6)
(15.2.5) and (15.2.6) can be simplified to the following forms

\[ s = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} \] for unclassified data

\[ = \sqrt{\frac{\sum f_i x_i^2}{N} - \bar{x}^2} \] for a grouped frequency distribution.

............................. (15.2.7)

Sometimes the square of standard deviation, known as variance, is regarded as a measure of dispersion. We have, then,

\[ \text{Variance} = s^2 = \frac{\sum (x_i - \bar{x})^2}{n} \] for unclassified data

\[ = \frac{\sum f_i (x_i - \bar{x})^2}{N} \] for a grouped frequency distribution..........................(15.2.8)

A relative measure of dispersion using standard deviation is given by coefficient of variation (cv) which is defined as the ratio of standard deviation to the corresponding arithmetic mean, expressed as a percentage.

\[ \text{Coefficient of Variation (CV)} = \frac{\text{SD}}{\text{AM}} \times 100 \]................................. (15.2.9)

**ILLUSTRATIONS:**

**Example 15.2.9:** Find the standard deviation and the coefficient of variation for the following numbers: 5, 8, 9, 2, 6

**Solution:** We present the computation in the following table.

<table>
<thead>
<tr>
<th>(x_i)</th>
<th>(x_i^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>(\sum x_i^2 = 210)</td>
</tr>
</tbody>
</table>

Applying (15.2.7), we get the standard deviation as

\[ s = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} \]

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\[
\begin{align*}
\text{STATISTICS} \\
= \sqrt{\frac{210}{5} - \left(\frac{30}{5}\right)^2} \quad \left(\text{since } \bar{x} = \frac{\Sigma x_i}{n}\right) \\
= \sqrt{42 - 36} \\
= \sqrt{6} \\
= 2.45 \\
\end{align*}
\]

The coefficient of variation is

\[
\text{CV} = 100 \times \frac{\text{SD}}{\text{AM}} \\
= 100 \times \frac{2.45}{6} \\
= 40.83
\]

**Example 15.2.10:** Show that for any two numbers \(a\) and \(b\), standard deviation is given by \(\frac{|a-b|}{2}\).

**Solution:** For two numbers \(a\) and \(b\), AM is given by \(\bar{x} = \frac{a+b}{2}\)

The variance is

\[
\begin{align*}
\sigma^2 &= \frac{\Sigma (x_i - \bar{x})^2}{2} \\
&= \frac{(a - \frac{a+b}{2})^2 + (b - \frac{a+b}{2})^2}{2} \\
&= \frac{(a-b)^2 + (a-b)^2}{2} \\
&= \frac{2(a-b)^2}{4} \\
&= \frac{(a-b)^2}{2} \\
\end{align*}
\]

\[\Rightarrow \quad \sigma = \frac{|a-b|}{2}\]

(The absolute sign is taken, as SD cannot be negative).

**Example 15.2.11:** Prove that for the first \(n\) natural numbers, SD is \(\sqrt{\frac{n^2-1}{12}}\).
**Solution:** for the first \( n \) natural numbers AM is given by

\[
\bar{x} = \frac{1 + 2 + 3 + \ldots + n}{n}
\]

\[
= \frac{n(n + 1)}{2n}
\]

\[
= \frac{n + 1}{2}
\]

\[
\therefore \text{SD} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}
\]

\[
= \sqrt{\frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{n} - \left(\frac{n + 1}{2}\right)^2}
\]

\[
= \sqrt{\frac{n(n + 1)(2n + 1)}{6n} - \frac{(n + 1)^2}{4}}
\]

\[
= \sqrt{\frac{(n + 1)(4n^2 + 2n - 3n - 3)}{12}} = \sqrt{\frac{n^2 - 1}{12}}
\]

Thus, SD of first \( n \) natural numbers is \( \text{SD} = \sqrt{\frac{n^2 - 1}{12}} \)

We consider the following formula for computing standard deviation from grouped frequency distribution with a view to saving time and computational labour:

\[
S = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}
\]

\[
\text{Where } d_i = \frac{x_i - A}{C}
\]

**Example 15.2.12:** Find the SD of the following distribution:

<table>
<thead>
<tr>
<th>Weight (kgs.)</th>
<th>50-52</th>
<th>52-54</th>
<th>54-56</th>
<th>56-58</th>
<th>58-60</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Students</td>
<td>17</td>
<td>35</td>
<td>28</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>

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Solution:

Table 15.2.7
Computation of SD

<table>
<thead>
<tr>
<th>Weight (kgs.) (1)</th>
<th>No. of Students (f) (2)</th>
<th>Mid-value (xᵢ) (3)</th>
<th>dᵢ=xᵢ – 55 (4)</th>
<th>fᵢdᵢ (5)=(2)×(4)</th>
<th>fᵢdᵢ² (6)=(5)×(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50-52</td>
<td>17</td>
<td>51</td>
<td>–2</td>
<td>–34</td>
<td>68</td>
</tr>
<tr>
<td>52-54</td>
<td>35</td>
<td>53</td>
<td>–1</td>
<td>–35</td>
<td>35</td>
</tr>
<tr>
<td>54-56</td>
<td>28</td>
<td>55</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>56-58</td>
<td>15</td>
<td>57</td>
<td>1</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>58-60</td>
<td>5</td>
<td>59</td>
<td>2</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>–</td>
<td>–</td>
<td>–44</td>
<td>138</td>
</tr>
</tbody>
</table>

Applying (11.33), we get the SD of weight as

\[
\sqrt{\frac{\sum fᵢdᵢ²}{N}} - \left(\frac{\sum fᵢdᵢ}{N}\right)^2 \times C
\]

\[
= \sqrt{\frac{138}{100}} - \left(\frac{-44}{100}\right)^2 \times 2 \text{kgs.}
\]

\[
= \sqrt{1.38 - 0.1936 \times 2} \text{kgs.}
\]

\[
= 2.18 \text{kgs.}
\]

Properties of standard deviation

I. If all the observations assumed by a variable are constant i.e. equal, then the SD is zero. This means that if all the values taken by a variable x is k, say, then s = 0. This result applies to range as well as mean deviation.

II. SD remains unaffected due to a change of origin but is affected in the same ratio due to a change of scale i.e., if there are two variables x and y related as y = a+bx for any two constants a and b, then SD of y is given by

\[
s_y = |b|s_x
\]

III. If there are two groups containing n₁ and n₂ observations, \( \bar{x}_1 \) and \( \bar{x}_2 \) as respective AM’s, \( s_1 \) and \( s_2 \) as respective SD’s , then the combined SD is given by

\[
s = \sqrt{\frac{n₁s₁² + n₂s₂² + n₁d₁² + n₂d₂²}{n₁ + n₂}}
\]
MEASURES OF CENTRAL TENDENCY AND DISPERSION

where, \( d_1 = \bar{x}_1 - \bar{x} \)
\( d_2 = \bar{x}_2 - \bar{x} \)

and \( \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \) = combined AM

This result can be extended to more than 2 groups. For \( x(72) \) groups, we have

\[
\sigma = \sqrt{\frac{\sum n_i \sigma_i^2 + \sum n_i d_i^2}{\sum n_i}} \quad \text{......................... (15.2.13)}
\]

With \( d_i = x_i - \bar{x} \)

and \( \bar{x} = \frac{\sum n_i \bar{x}_i}{\sum n_i} \)

Where \( \bar{x}_i = \bar{x}_2 (11.35) \) is reduced to

\[
\sigma = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2}}
\]

**Example 15.2.13:** If AM and coefficient of variation of \( x \) are 10 and 40 respectively, what is the variance of \((15–2x)\)?

**Solution:** let \( y = 15 – 2x \)

Then applying (11.34), we get,

\[
s_y = 2 \times s_x \quad \text{................................. (1)}
\]

As given \( cv_x = \text{coefficient of variation of } x = 40 \) and \( \bar{x} = 10 \)

This \( cv_x = \frac{s_x}{x} \times 100 \)

\[
40 = \frac{s_x}{10} \times 100
\]

\[
\Rightarrow s_x = 4
\]

From (1), \( s_y = 2 \times 4 = 8 \)

Therefore, variance of \((15–2x) = s_y^2 = 64 \)

**Example 15.2.14:** Compute the SD of 9, 5, 8, 6, 2.

Without any more computation, obtain the SD of

Sample I \(-1, -5, -2, -4, -8,\)
Sample II \(90, 50, 80, 60, 20,\)
Sample III \(23, 15, 21, 17, 9.\)
Solution:

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$x_i^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td><strong>30</strong></td>
<td><strong>210</strong></td>
</tr>
</tbody>
</table>

The SD of the original set of observations is given by

$$ s = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} $$

$$ = \sqrt{\frac{210}{5} - \left(\frac{30}{5}\right)^2} $$

$$ = \sqrt{42 - 36} $$

$$ = \sqrt{6} $$

$$ = 2.45 $$

If we denote the original observations by $x$ and the observations of sample I by $y$, then we have

$$ y = -10 + x $$

$$ y = (10) + (1) x $$

$$ \therefore S_y = || \times S_x $$

$$ = 1 \times 2.45 $$

$$ = 2.45 $$

In case of sample II, $x$ and $y$ are related as

$$ Y = 10x $$

$$ = 0 + (15)x $$

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\[ s_y = 10 \times s_x \]
\[ = 10 \times 2.45 \]
\[ = 24.50 \]

And lastly, \( y = (5) + (2)x \)
\[ \Rightarrow s_y = 2 \times 2.45 \]
\[ = 4.90 \]

**Example 15.2.15:** For a group of 60 boy students, the mean and SD of stats. marks are 45 and 2 respectively. The same figures for a group of 40 girl students are 55 and 3 respectively. What is the mean and SD of marks if the two groups are pooled together?

**Solution:**

As given \( n_1 = 60, \bar{x}_1 = 45, s_1 = 2 \) \( n_2 = 40, \bar{x}_2 = 55, s_2 = 3 \)

Thus the combined mean is given by

\[ \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \]

\[ = \frac{60 \times 45 + 40 \times 55}{60 + 40} \]
\[ = 49 \]

Thus \( d_1 = \bar{x}_1 - \bar{x} = 45 - 49 = -4 \)
\( d_2 = \bar{x}_2 - \bar{x} = 55 - 49 = 6 \)

Applying (11.35), we get the combined SD as

\[ s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}} \]

\[ s = \sqrt{\frac{60 \times 2^2 + 40 \times 3^2 + 60 \times (-4)^2 + 40 \times 6^2}{60 + 40}} \]
\[ = \sqrt{30} \]
\[ = 5.48 \]

**Example 15.2.16:** The mean and standard deviation of the salaries of the two factories are provided below:

<table>
<thead>
<tr>
<th>Factory</th>
<th>No. of Employees</th>
<th>Mean Salary</th>
<th>SD of Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
<td>₹ 4800</td>
<td>₹ 10</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>₹ 5000</td>
<td>₹ 12</td>
</tr>
</tbody>
</table>

i) Find the combined mean salary and standard deviation of salary.

ii) Examine which factory has more consistent structure so far as satisfying its employees are concerned.

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Solution: Here we are given
\[ n_1 = 30, \quad \bar{x}_1 = 4800, \quad s_1 = 10, \]
\[ n_2 = 20, \quad \bar{x}_2 = 5000, \quad s_2 = 12 \]

i) \[ \frac{30 \times 4800 + 20 \times 5000}{30 + 20} = 4800 \]
\[ d_1 = \bar{x}_1 - \bar{x} = 4800 - 4880 = -80 \]
\[ d_2 = \bar{x}_2 - \bar{x} = 5000 - 4880 = 120 \]

hence, the combined SD in rupees is given by
\[ s = \sqrt{\frac{30\times10^2 + 20\times12^2 + 30\times(-80)^2 + 20\times120^2}{30 + 20}} \]
\[ = \sqrt{9717.60} \]
\[ = 98.58 \]

thus the combined mean salary and the combined standard deviation of salary are ₹ 4880 and ₹ 98.58 respectively.

ii) In order to find the more consistent structure, we compare the coefficients of variation of the two factories. Letting \( CV_A = 100 \times \frac{S_A}{\bar{x}_A} \) and \( CV_B = 100 \times \frac{S_B}{\bar{x}_B} \)

We would say factory A is more consistent if \( CV_A < CV_B \). Otherwise factory B would be more consistent.

Now \( CV_A = 100 \times \frac{S_A}{\bar{x}_A} = 100 \times \frac{10}{4800} = 0.21 \)

and \( CV_B = 100 \times \frac{S_B}{\bar{x}_B} = 100 \times \frac{12}{5000} = 0.24 \)

Thus we conclude that factory A has more consistent structure.

Example 15.2.17: A student computes the AM and SD for a set of 100 observations as 50 and 5 respectively. Later on, she discovers that she has made a mistake in taking one observation as 60 instead of 50. What would be the correct mean and SD if

i) The wrong observation is left out?
ii) The wrong observation is replaced by the correct observation?

Solution: As given, \( n = 100, \quad \bar{x} = 50, \quad S = 5 \)
Wrong observation = 60(x), correct observation = 50(V)

\[ \bar{x} = \frac{\sum x_i}{n} \]

\[ \Rightarrow \sum x_i = n\bar{x} = 100 \times 50 = 5000 \]

and \[ s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 \]

\[ \Rightarrow \sum x_i^2 = n(\bar{x}^2 + s^2) = 100(50^2 + 5^2) = 252500 \]

i) Sum of the 99 observations = 5000 – 60 = 4940

AM after leaving the wrong observation = 4940/99 = 49.90

Sum of squares of the observation after leaving the wrong observation
= 252500 – 60^2 = 248900

Variance of the 99 observations = 248900/99 – (49.90)^2
= 2514.14 – 2490.01
= 24.13

\[ \therefore \text{SD of 99 observations} = 4.91 \]

ii) Sum of the 100 observations after replacing the wrong observation by the correct observation
= 5000 – 60 + 50 = 4990

\[ \text{AM} = \frac{4990}{100} = 49.90 \]

Corrected sum of squares = 252500 + 50^2 – 60^2 = 251400

Corrected SD
= \[ \sqrt{\frac{251400}{100} - (49.90)^2} = \sqrt{23.94} = 4.90 \]

15.2.5 QUARTILE DEVIATION

Another measure of dispersion is provided by quartile deviation or semi-inter-quartile range which is given by

\[ Q_d = \frac{Q_3 - Q_1}{2} \] .................................(15.2.14)

A relative measure of dispersion using quartiles is given by coefficient of quartile deviation which is

\[ \text{Coefficient of quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100 \] .................................(15.2.15)
Quartile deviation provides the best measure of dispersion for open-end classification. It is also less affected due to sampling fluctuations. Like other measures of dispersion, quartile deviation remains unaffected due to a change of origin but is affected in the same ratio due to change in scale.

**Example 15.2.18:** Following are the marks of the 10 students: 56, 48, 65, 35, 42, 75, 82, 60, 55, 50. Find quartile deviation and also its coefficient.

**Solution:**
After arranging the marks in an ascending order of magnitude, we get 35, 42, 48, 50, 55, 56, 60, 65, 75, 82

First quartile \( (Q_1) = \frac{(n + 1)}{4} \) th observation

\[
= \frac{(10 + 1)}{4} = 2.75 \text{ th observation}
= 2\text{nd observation} + 0.75 \times \text{difference between the third and the 2\text{nd observation}}.
= 42 + 0.75 \times (48 - 42)
= 46.50
\]

Third quartile \( (Q_3) = \frac{3(n + 1)}{4} \) th observation

\[
= 8.25 \text{ th observation}
= 65 + 0.25 \times 10
= 67.50
\]

Thus applying (11.37), we get the quartile deviation as

\[
\frac{Q_3 - Q_1}{2} = \frac{67.50 - 46.50}{2} = 10.50
\]

Also, using (11.38), the coefficient of quartile deviation

\[
= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100
= \frac{67.50 - 46.50}{67.50 + 46.50} \times 100
= 18.42
\]

**Example 15.2.19:** If the quartile deviation of \( x \) is 6 and \( 3x + 6y = 20 \), what is the quartile deviation of \( y \)?
**Solution:**  
\[3x + 6y = 20\]

\[\Rightarrow y = \left(\frac{20}{6}\right) + \left(\frac{-3}{6}\right)x\]

Therefore, quartile deviation of \(y = \frac{1}{6} \times \text{quartile deviation of } x\)

\[= \frac{1}{2} \times 6 = 3.\]

**Example 15.2.20:** Find an appropriate measures of dispersion from the following data:

<table>
<thead>
<tr>
<th>Daily wages (₹)</th>
<th>upto 20</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
<th>80-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of workers</td>
<td>5</td>
<td>11</td>
<td>14</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

**Solution:** Since this is an open-end classification, the appropriate measure of dispersion would be quartile deviation as quartile deviation does not take into account the first twenty five percent and the last twenty five per cent of the observations.

**Table 15.2.8**

<table>
<thead>
<tr>
<th>Daily wages in (₹) (Class boundary)</th>
<th>No. of workers (less than cumulative frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>40</td>
<td>16</td>
</tr>
<tr>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td>80</td>
<td>37</td>
</tr>
<tr>
<td>100</td>
<td>40</td>
</tr>
</tbody>
</table>

Here \(a\) denotes the first Class Boundary

\[Q_1 = ₹ \left[20 + \frac{10 - 5}{16 - 5} \times 20\right] = ₹ 29.09\]

\[Q_3 = ₹ 60\]

Thus quartile deviation of wages is given by

\[\frac{Q_3 - Q_1}{2} = \frac{₹ 60 - ₹ 29.09}{2} = ₹ 15.46\]
**Example 15.2.21:** The mean and variance of 5 observations are 4.80 and 6.16 respectively. If three of the observations are 2, 3 and 6, what are the remaining observations?

**Solution:** Let the remaining two observations be $a$ and $b$, then as given

\[
\frac{2 + 3 + 6 + a + b}{5} = 4.80
\]

\[
\Rightarrow 11 + a + b = 24
\]

\[
\Rightarrow a + b = 13 \quad \text{.................(1)}
\]

and

\[
\frac{2^2 + a^2 + b^2 + 3^2 + 6^2 - (4.80)^2}{5}
\]

\[
\Rightarrow \frac{49 + a^2 + b^2}{5} - 23.04 = 6.16
\]

\[
\Rightarrow 49 + a^2 + b^2 = 146
\]

\[
\Rightarrow a^2 + b^2 = 97 \quad \text{.................(2)}
\]

From (1), we get $a = 13 - b \quad \text{...........(3)}$

Eliminating $a$ from (2) and (3), we get

\[
(13 - b)^2 + b^2 = 97
\]

\[
\Rightarrow 169 - 26b + 2b^2 = 97
\]

\[
\Rightarrow b^2 - 13b + 36 = 0
\]

\[
\Rightarrow (b-4)(b-9) = 0
\]

\[
\Rightarrow b = 4 \text{ or } 9
\]

From (3), $a = 9 \text{ or } 4$

Thus the remaining observations are 4 and 9.

**Example 15.2.22:** After shift of origin and change of scale, a frequency distribution of a continuous variable with equal class length takes the following form of the changed variable $d$:

<table>
<thead>
<tr>
<th>$d$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>17</td>
<td>35</td>
<td>28</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>

If the mean and standard deviation of the original frequency distribution are 54.12 and 2.1784 respectively, find the original frequency distribution.

**Solution:** We need find out the origin $A$ and scale $C$ from the given conditions.

Since \(d_i = \frac{x_i - A}{C}\)

\[
\Rightarrow x_i = A + Cd_i
\]
Once A and C are known, the mid-values $x_i$’s would be known. Finally, we convert the mid-values to the corresponding class boundaries by using the formula:

- LCB = $x_i - C/2$
- UCB = $x_i + C/2$

On the basis of the given data, we find that

\[ \sum f_i d_i = -44, \quad \sum f_i d_i^2 = 138 \text{ and } N = 100 \]

Hence

\[ s = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times C \]

\[ \Rightarrow 2.1784 = \sqrt{\frac{138}{100} - \left(\frac{-44}{100}\right)^2} \times C \]

\[ \Rightarrow 2.1784 = \sqrt{1.38 - 0.1936} \times C \]

\[ \Rightarrow 2.1784 = 1.0892 \times C \]

\[ \Rightarrow C = 2 \]

Further,

\[ \bar{x} = A + \frac{\sum f_i d_i}{N} \times C \]

\[ \Rightarrow 54.12 = A + \frac{-44}{100} \times 2 \]

\[ \Rightarrow 54.12 = A - 0.88 \]

\[ \Rightarrow A = 55 \]

Thus

\[ x_i = A + C d_i \]

\[ \Rightarrow x_i = 55 + 2d_i \]

<table>
<thead>
<tr>
<th>$d_i$</th>
<th>$f_i$</th>
<th>$x_i = 55 + 2d_i$</th>
<th>Class interval $x_i \pm \frac{C}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>17</td>
<td>51</td>
<td>50-52</td>
</tr>
<tr>
<td>-1</td>
<td>35</td>
<td>53</td>
<td>52-54</td>
</tr>
<tr>
<td>0</td>
<td>28</td>
<td>55</td>
<td>54-56</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>57</td>
<td>56-58</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>59</td>
<td>58-60</td>
</tr>
</tbody>
</table>
Example 15.2.23: Compute coefficient of variation from the following data:

<table>
<thead>
<tr>
<th>Age</th>
<th>under 10</th>
<th>under 20</th>
<th>under 30</th>
<th>under 40</th>
<th>under 50</th>
<th>under 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of persons dying</td>
<td>10</td>
<td>18</td>
<td>30</td>
<td>45</td>
<td>60</td>
<td>80</td>
</tr>
</tbody>
</table>

Solution: What is given in this problem is less than cumulative frequency distribution. We need first convert it to a frequency distribution and then compute the coefficient of variation.

<table>
<thead>
<tr>
<th>Age class Interval</th>
<th>No. of persons dying ((f_i))</th>
<th>No. of persons</th>
<th>Mid-value (x_i)</th>
<th>(d_i = \frac{x_i - 25}{10})</th>
<th>(f_i d_i)</th>
<th>(f_i d_i^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>10</td>
<td>5</td>
<td>-2</td>
<td>-20</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>10-20</td>
<td>18-10= 8</td>
<td>15</td>
<td>-1</td>
<td>-8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>20-30</td>
<td>30-18=12</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>30-40</td>
<td>45-30=15</td>
<td>35</td>
<td>1</td>
<td>15</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>40-50</td>
<td>60-45=15</td>
<td>45</td>
<td>2</td>
<td>30</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>50-60</td>
<td>80-60=20</td>
<td>55</td>
<td>3</td>
<td>60</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>-</td>
<td>-</td>
<td>77</td>
<td>303</td>
<td></td>
</tr>
</tbody>
</table>

The AM is given by:

\[
\bar{x} = A + \frac{\sum f_i d_i}{N} \times C
\]

\[
= \left(25 + \frac{77}{80} \times 10\right) \text{ years}
\]

\[
= 34.63 \text{ years}
\]

The standard deviation is

\[
s = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times C
\]

\[
= \sqrt{\frac{303}{80} - \left(\frac{77}{80}\right)^2} \times 10 \text{ years}
\]
15.53

MEASURES OF CENTRAL TENDENCY AND DISPERSION

\[
= \sqrt{3.79 - 0.93 \times 10} \text{ years} \\
= 16.91 \text{ years}
\]

Thus the coefficient of variation is given by

\[
CV = \frac{S}{X} \times 100 \\
= \frac{16.91}{34.63} \times 100 \\
= 48.83
\]

Example 15.2.24: You are given the distribution of wages in two factors A and B

<table>
<thead>
<tr>
<th>Wages in ₹</th>
<th>100-200</th>
<th>200-300</th>
<th>300-400</th>
<th>400-500</th>
<th>500-600</th>
<th>600-700</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of workers in A</td>
<td>8</td>
<td>12</td>
<td>17</td>
<td>10</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>No. of workers in B</td>
<td>6</td>
<td>18</td>
<td>25</td>
<td>12</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

State in which factory, the wages are more variable.

Solution:

As explained in example 11.36, we need compare the coefficient of variation of A (i.e. \(v_A\)) and of B (i.e. \(v_B\)).

If \(v_A > v_B\), then the wages of factory A would be more variable. Otherwise, the wages of factory B would be more variable where

\[
V_A = 100 \times \frac{S_A}{X_A} \quad \text{and} \quad V_B = 100 \times \frac{S_B}{X_B}
\]

<table>
<thead>
<tr>
<th>Wages in rupees</th>
<th>Mid-value</th>
<th>d=</th>
<th>No. of workers of A (f_A)</th>
<th>No. of workers of B (f_B)</th>
<th>(f_Ad)</th>
<th>(f_Ad^2)</th>
<th>(f_Bd)</th>
<th>(f_Bd^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
</tr>
<tr>
<td>100-200</td>
<td>150</td>
<td>−2</td>
<td>8</td>
<td>6</td>
<td>−16</td>
<td>32</td>
<td>−12</td>
<td>24</td>
</tr>
<tr>
<td>200-300</td>
<td>250</td>
<td>−1</td>
<td>12</td>
<td>18</td>
<td>−12</td>
<td>12</td>
<td>−18</td>
<td>18</td>
</tr>
<tr>
<td>300-400</td>
<td>350</td>
<td>0</td>
<td>17</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>400-500</td>
<td>450</td>
<td>1</td>
<td>10</td>
<td>12</td>
<td>10</td>
<td>10</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>500-600</td>
<td>550</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>600-700</td>
<td>650</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>Total</td>
<td>–</td>
<td>–</td>
<td>50</td>
<td>65</td>
<td>−11</td>
<td>71</td>
<td>−8</td>
<td>80</td>
</tr>
</tbody>
</table>

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For Factory A

\[ \bar{x}_A = \frac{350 + \frac{-11}{50} \times 100}{\frac{50}{71}} = \text{Rs. 328} \]

\[ S_A = \sqrt{\frac{71}{50} \left( \frac{-11}{50} \right)^2 \times 100} = \text{Nu.117.12} \]

\[ : V_A = \frac{S_A}{\bar{x}_A} \times 100 = 35.71 \]

For Factory B

\[ \bar{x}_B = \frac{350 + \frac{-8}{65} \times 100}{\frac{65}{80}} = \text{Rs. 337.69} \]

\[ S_B = \sqrt{\frac{80}{65} \left( \frac{-8}{65} \right)^2 \times 100} = \text{Rs. 110.25} \]

\[ : V_B = \frac{110.25}{337.69} \times 100 = 32.65 \]

As \( V_A > V_B \), the wages for factory A is more variable.

**SUMMARY**

- All these merits of standard deviation make SD as the most widely and commonly used measure of dispersion.
- Range is the quickest to compute and as such, has its application in statistical quality control. However, range is based on only two observations and affected too much by the presence of extreme observation(s).
- Mean deviation is rigidly defined, based on all the observations and not much affected by sampling fluctuations. However, mean deviation is difficult to comprehend and its computation is also time consuming and laborious. Furthermore, unlike SD, mean deviation does not possess mathematical properties.
- Quartile deviation is also rigidly defined, easy to compute and not much affected by sampling fluctuations. The presence of extreme observations has no impact on quartile deviation since quartile deviation is based on the central fifty-percent of the observations. However, quartile deviation is not based on all the observations and it has no desirable mathematical properties. Nevertheless, quartile deviation is the best measure of dispersion for open-end classifications.
EXERCISE — UNIT-II

Set A

Write down the correct answers. Each question carries one mark.

1. Which of the following statements is correct?
   (a) Two distributions may have identical measures of central tendency and dispersion.
   (b) Two distributions may have the identical measures of central tendency but different measures of dispersion.
   (c) Two distributions may have the different measures of central tendency but identical measures of dispersion.
   (d) All the statements (a), (b) and (c).

2. Dispersion measures
   (a) The scatterness of a set of observations
   (b) The concentration of a set of observations
   (c) Both (a) and (b)
   (d) Neither (a) and (b).

3. When it comes to comparing two or more distributions we consider
   (a) Absolute measures of dispersion
   (b) Relative measures of dispersion
   (c) Both (a) and (b)
   (d) Either (a) or (b).

4. Which one is easier to compute?
   (a) Relative measures of dispersion
   (b) Absolute measures of dispersion
   (c) Both (a) and (b)
   (d) Range

5. Which one is an absolute measure of dispersion?
   (a) Range
   (b) Mean Deviation
   (c) Standard Deviation
   (d) All these measures

6. Which measure of dispersion is most useful?
   (a) Standard deviation
   (b) Quartile deviation
   (c) Mean deviation
   (d) Range

7. Which measures of dispersions is not affected by the presence of extreme observations?
   (a) Range
   (b) Mean deviation
   (c) Standard deviation
   (d) Quartile deviation

8. Which measure of dispersion is based on the absolute deviations only?
   (a) Standard deviation
   (b) Mean deviation
   (c) Quartile deviation
   (d) Range

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9. Which measure is based on only the central fifty percent of the observations?
   (a) Standard deviation  (b) Quartile deviation
   (c) Mean deviation      (d) All these measures

10. Which measure of dispersion is based on all the observations?
    (a) Mean deviation      (b) Standard deviation
    (c) Quartile deviation  (d) (a) and (b) but not (c)

11. The appropriate measure of dispersion for open-end classification is
    (a) Standard deviation  (b) Mean deviation
    (c) Quartile deviation  (d) All these measures.

12. The most commonly used measure of dispersion is
    (a) Range              (b) Standard deviation
    (c) Coefficient of variation  (d) Quartile deviation.

13. Which measure of dispersion has some desirable mathematical properties?
    (a) Standard deviation  (b) Mean deviation
    (c) Quartile deviation  (d) All these measures

14. If the profits of a company remains the same for the last ten months, then the standard
deviation of profits for these ten months would be?
    (a) Positive  (b) Negative  (c) Zero  (d) (a) or (c)

15. Which measure of dispersion is considered for finding a pooled measure of dispersion after
    combining several groups?
    (a) Mean deviation  (b) Standard deviation
    (c) Quartile deviation (d) Any of these

16. A shift of origin has no impact on
    (a) Range  (b) Mean deviation
    (c) Standard deviation  (d) All these and quartile deviation.

17. The range of 15, 12, 10, 9, 17, 20 is
    (a) 5     (b) 12     (c) 13     (d) 11.

18. The standard deviation of 10, 16, 10, 16, 10, 16, 16 is
    (a) 4     (b) 6     (c) 3     (d) 0.

19. For any two numbers SD is always
    (a) Twice the range  (b) Half of the range
    (c) Square of the range  (d) None of these.
20. If all the observations are increased by 10, then
   (a) SD would be increased by 10
   (b) Mean deviation would be increased by 10
   (c) Quartile deviation would be increased by 10
   (d) All these three remain unchanged.

21. If all the observations are multiplied by 2, then
   (a) New SD would be also multiplied by 2
   (b) New SD would be half of the previous SD
   (c) New SD would be increased by 2
   (d) New SD would be decreased by 2.

Set B

Write down the correct answers. Each question carries two marks.

1. What is the coefficient of range for the following wages of 8 workers?
   ₹ 80, ₹ 65, ₹ 90, ₹ 60, ₹ 75, ₹ 70, ₹ 72, ₹ 85.
   (a) ₹ 30   (b) ₹ 20   (c) 30   (d) 20

2. If \( R_x \) and \( R_y \) denote ranges of \( x \) and \( y \) respectively where \( x \) and \( y \) are related by \( 3x+2y+10=0 \), what would be the relation between \( x \) and \( y \)?
   (a) \( R_x = R_y \)   (b) \( 2 R_x = 3 R_y \)   (c) \( 3 R_x = 2 R_y \)   (d) \( R_x = 2 R_y \)

3. What is the coefficient of range for the following distribution?
   Class Interval : 10-19 20-29 30-39 40-49 50-59
   Frequency: 11 25 16 7 3
   (a) 22   (b) 50   (c) 72.46   (d) 75.82

4. If the range of \( x \) is 2, what would be the range of \(-3x + 50\) ?
   (a) 2   (b) 6   (c) –6   (d) 44

5. What is the value of mean deviation about mean for the following numbers?
   5, 8, 6, 3, 4.
   (a) 5.20   (b) 7.20   (c) 1.44   (d) 2.23

6. What is the value of mean deviation about mean for the following observations?
   50, 60, 50, 50, 60, 60, 60, 50, 50, 60, 60, 60, 60, 60, 50.
   (a) 5   (b) 7   (c) 35   (d) 10

7. The coefficient of mean deviation about mean for the first 9 natural numbers is
   (a) \( \frac{200}{9} \)   (b) 80   (c) \( \frac{400}{9} \)   (d) 50.
8. If the relation between \( x \) and \( y \) is \( 5y - 3x = 10 \) and the mean deviation about mean for \( x \) is 12, then the mean deviation of \( y \) about mean is
   
   (a) 7.20 (b) 6.80 (c) 20 (d) 18.80.

9. If two variables \( x \) and \( y \) are related by \( 2x + 3y - 7 = 0 \) and the mean and mean deviation about mean of \( x \) are 1 and 0.3 respectively, then the coefficient of mean deviation of \( y \) about its mean is
   
   (a) –5 (b) 12 (c) 50 (d) 4.

10. The mean deviation about mode for the numbers \( \frac{4}{11}, \frac{6}{11}, \frac{8}{11}, \frac{9}{11}, \frac{12}{11}, \frac{8}{11} \) is
    
    (a) 1/6 (b) 1/11 (c) 6/11 (d) 5/11.

11. What is the standard deviation of 5, 5, 9, 9, 9, 10, 5, 10, 10?
    
    (a) \( \sqrt{14} \) (b) \( \frac{\sqrt{42}}{3} \) (c) 4.50 (d) 8

12. If the mean and SD of \( x \) are \( a \) and \( b \) respectively, then the SD of \( \frac{x-a}{b} \) is
    
    (a) –1 (b) 1 (c) \( ab \) (d) \( a/b \).

13. What is the coefficient of variation of the following numbers?
    53, 52, 61, 60, 64.
    
    (a) 8.09 (b) 18.08 (c) 20.23 (d) 20.45

14. If the SD of \( x \) is 3, what is the variance of \( (5 - 2x) \)?
    
    (a) 36 (b) 6 (c) 1 (d) 9

15. If \( x \) and \( y \) are related by \( 2x + 3y + 4 = 0 \) and SD of \( x \) is 6, then SD of \( y \) is
    
    (a) 22 (b) 4 (c) \( \sqrt{5} \) (d) 9.

16. The quartiles of a variable are 45, 52 and 65 respectively. Its quartile deviation is
    
    (a) 10 (b) 20 (c) 25 (d) 8.30.

17. If \( x \) and \( y \) are related as \( 3x + 4y = 20 \) and the quartile deviation of \( x \) is 12, then the quartile deviation of \( y \) is
    
    (a) 16 (b) 14 (c) 10 (d) 9.

18. If the SD of the 1st \( n \) natural numbers is 2, then the value of \( n \) must be
    
    (a) 2 (b) 7 (c) 6 (d) 5.

19. If \( x \) and \( y \) are related by \( y = 2x + 5 \) and the SD and AM of \( x \) are known to be 5 and 10 respectively, then the coefficient of variation is
    
    (a) 25 (b) 30 (c) 40 (d) 20.

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20. The mean and SD for a, b and 2 are 3 and \( \frac{2}{\sqrt{3}} \) respectively. The value of ab would be

(a) 5  
(b) 6  
(c) 11  
(d) 3.

Set C

Write down the correct answer. Each question carries 5 marks.

1. What is the mean deviation about mean for the following distribution?
   - Variable: 5 10 15 20 25 30
   - Frequency: 3 4 6 5 3 2
   - (a) 6.00  
     (b) 5.93  
     (c) 6.07  
     (d) 7.20

2. What is the mean deviation about median for the following data?
   - X: 3 5 7 9 11 13 15
   - F: 2 8 9 16 14 7 4
   - (a) 2.50  
     (b) 2.46  
     (c) 2.43  
     (d) 2.37

3. What is the coefficient of mean deviation for the following distribution of heights? Take deviation from AM.
   - Height in inches: 60-62 63-65 66-68 69-71 72-74
   - No. of students: 5 22 28 17 3
   - (a) 2.30 inches  
     (b) 3.45 inches  
     (c) 3.82 inches  
     (d) 2.48 inches

4. The mean deviation of weights about median for the following data:
   - Weight (lb): 131-140 141-150 151-160 161-170 171-180 181-190
   - No. of persons: 3 8 13 15 6 5
   - Is given by
   - (a) 10.97  
     (b) 8.23  
     (c) 9.63  
     (d) 11.45.

5. What is the standard deviation from the following data relating to the age distribution of 200 persons?
   - Age (year): 20 30 40 50 60 70 80
   - No. of people: 13 28 31 46 39 23 20
   - (a) 15.29  
     (b) 16.87  
     (c) 18.00  
     (d) 17.52

6. What is the coefficient of variation for the following distribution of wages?
   - No. of workers: 17 28 21 15 13 6
   - (a) ₹ 14.73  
     (b) 14.73  
     (c) 26.93  
     (d) 20.82

7. Which of the following companies A and B is more consistent so far as the payment of dividend is concerned?

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8. The mean and SD for a group of 100 observations are 65 and 7.03 respectively. If 60 of these observations have mean and SD as 70 and 3 respectively, what is the SD for the group comprising 40 observations?

(a) 16 (b) 25 (c) 4 (d) 2

9. If two samples of sizes 30 and 20 have means as 55 and 60 and variances as 16 and 25 respectively, then what would be the SD of the combined sample of size 50?

(a) 5.00 (b) 5.06 (c) 5.23 (d) 5.35

10. The mean and SD of a sample of 100 observations were calculated as 40 and 5.1 respectively by a CA student who took one of the observations as 50 instead of 40 by mistake. The current value of SD would be

(a) 4.90 (b) 5.00 (c) 4.88 (d) 4.85.

11. The value of appropriate measure of dispersion for the following distribution of daily wages

<table>
<thead>
<tr>
<th>Wages (₹)</th>
<th>Below 30</th>
<th>30-39</th>
<th>40-49</th>
<th>50-59</th>
<th>60-79</th>
<th>Above 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of workers</td>
<td>5</td>
<td>7</td>
<td>18</td>
<td>32</td>
<td>28</td>
<td>10</td>
</tr>
</tbody>
</table>

is given by

(a) ₹ 11.03 (b) ₹ 10.50 (c) 11.68 (d) ₹ 11.68.

UNIT-II: ANSWERS

Set A

1. (d) 2. (a) 3. (b) 4. (d) 5. (d) 6. (a)
7. (d) 8. (b) 9. (b) 10. (d) 11. (c) 12. (b)
13. (a) 14. (c) 15. (b) 16. (d) 17. (d) 18. (c)
19. (b) 20. (d) 21. (a)

Set B

1. (d) 2. (c) 3. (c) 4. (b) 5. (c) 6. (a)
7. (c) 8. (a) 9. (b) 10. (b) 11. (b) 12. (b)
13. (a) 14. (a) 15. (b) 16. (a) 17. (d) 18. (b)
19. (c) 20. (a)

Set C

1. (c) 2. (d) 3. (b) 4. (a) 5. (b) 6. (c)
7. (a) 8. (c) 9. (b) 10. (b) 11. (a)
ADDITIONAL QUESTION BANK

1. The number of measures of central tendency is
   (a) two (b) three (c) four (d) five

2. The words “mean” or “average” only refer to
   (a) A.M (b) G.M (c) H.M (d) none

3. _________ is the most stable of all the measures of central tendency.
   (a) G.M (b) H.M (c) A.M (d) none.

4. Mean is of _______ types.
   (a) 3 (b) 4 (c) 8 (d) 5

5. Weighted A.M is related to
   (a) G.M (b) frequency (c) H.M (d) none.

6. Frequencies are also called weights.
   (a) True (b) false (c) both (d) none

7. The algebraic sum of deviations of observations from their A.M is
   (a) 2 (b) -1 (c) 1 (d) 0

8. G.M of a set of n observations is the _______ root of their product.
   (a) n/2 th (b) (n+1)th (c) nth (d) (n -1)th

9. The algebraic sum of deviations of 8, 1, 6 from the A.M viz.5 is
   (a) -1 (b) 0 (c) 1 (d) none

10. G.M of 8, 4,2 is
    (a) 4 (b) 2 (c) 8 (d) none

11. _________ is the reciprocal of the A.M of reciprocal of observations.
    (a) H.M (b) G.M (c) both (d) none

12. A.M is never less than G.M
    (a) True (b) false (c) both (d) none

13. G.M is less than H.M
    (a) true (b) false (c) both (d) none

14. The value of the middlemost item when they are arranged in order of magnitude is called
    (a) standard deviation (b) mean (c) mode (d) median

15. Median is unaffected by extreme values.
    (a) true (b) false (c) both (d) none

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16. Median of 2, 5, 8, 4, 9, 6, 71 is
   (a) 9  (b) 8  (c) 5  (d) 6

17. The value which occurs with the maximum frequency is called
   (a) median  (b) mode  (c) mean  (d) none

18. In the formula \( \text{Mode} = L_1 + \frac{d_1 \times c}{d_1 + d_2} \)
   \( d_1 \) is the difference of frequencies in the modal class & the ———————— class.
   (a) preceding  (b) following  (c) both  (d) none

19. In the formula \( \text{Mode} = L_1 + \frac{d_1 \times c}{d_2} \)
   \( d_2 \) is the difference of frequencies in the modal class & the ———————— class.
   (a) preceding  (b) succeeding  (c) both  (d) none

20. In formula of median for grouped frequency distribution \( N \) is
   (a) total frequency  (b) frequency density
   (c) frequency  (d) cumulative frequency

21. When all observations occur with equal frequency ————— does not exit.
   (a) median  (b) mode  (c) mean  (d) none

22. Mode of the observations 2, 5, 8, 4, 3, 4, 4, 5, 2, 4, 4 is
   (a) 3  (b) 2  (c) 5  (d) 4

23. For the observations 5, 3, 6, 3, 5, 10, 7, 2 there are —————— modes.
   (a) 2  (b) 3  (c) 4  (d) 5

24. ——————— of a set of observations is defined to be their sum, divided by the no. of observations.
   (a) H.M  (b) G.M  (c) A.M  (d) none

25. Simple average is sometimes called
   (a) weighted average  (b) unweighted average
   (c) relative average  (d) none

26. When a frequency distribution is given, the frequencies themselves treated as weights.
   (a) True  (b) false  (c) both  (d) none

27. Each value is considered only once for
   (a) simple average  (b) weighted average
   (c) both  (d) none

28. Each value is considered as many times as it occurs for
   (a) simple average  (b) weighted average
   (c) both  (d) none
29. Multiplying the values of the variable by the corresponding weights and then dividing the sum of products by the sum of weights is
(a) simple average (b) weighted average (c) both (d) none
30. Simple & weighted average are equal only when all the weights are equal.
(a) True (b) false (c) both (d) none
31. The word “average” used in “simple average” and “weighted average” generally refers to
(a) median (b) mode (c) A.M, G.M or H.M (d) none
32. _______ average is obtained on dividing the total of a set of observations by their number
(a) simple (b) weighted (c) both (d) none
33. Frequencies are generally used as
(a) range (b) weights (c) mean (d) none
34. The total of a set of observations is equal to the product of their number of observations and
the
(a) A.M (b) G.M (c) H.M (d) none
35. The total of the deviations of a set of observations from their A.M is always
(a) 0 (b) 1 (c) -1 (d) none
36. Deviation may be positive or negative or zero
(a) true (b) false (c) both (d) none
37. The sum of the squares of deviations of a set of observations has the smallest value, when
the deviations are taken from their
(a) A.M (b) H.M (c) G.M (d) none
38. For a given set of positive observations H.M is less than G.M
(a) true (b) false (c) both (d) none
39. For a given set of positive observations A.M is greater than G.M
(a) true (b) false (c) both (d) none
40. Calculation of G.M is more difficult than
(a) A.M (b) H.M (c) median (d) none
41. _______ has a limited use
(a) A.M (b) G.M (c) H.M (d) none
42. A.M of 8, 1, 6 is
(a) 5 (b) 6 (c) 4 (d) none
43. ________ can be calculated from a frequency distribution with open end intervals
   (a) Median    (b) Mean    (c) Mode    (d) none
44. The values of all items are taken into consideration in the calculation of
   (a) median    (b) mean    (c) mode    (d) none
45. The values of extreme items do not influence the average in case of
   (a) median    (b) mean    (c) mode    (d) none
46. In a distribution with a single peak and moderate skewness to the right, it is closer to the
   concentration of the distribution in case of
   (a) mean    (b) median    (c) both    (d) none
47. If the variables x & z are so related that \( z = ax + b \) for each \( x = x_i \) where \( a \) & \( b \) are
   constants, then \( \bar{z} = a \bar{x} + b \)
   (a) true    (b) false    (c) both    (d) none
48. G.M is defined only when
   (a) all observations have the same sign and none is zero
   (b) all observations have the different sign and none is zero
   (c) all observations have the same sign and one is zero
   (d) all observations have the different sign and one is zero
49. ________ is useful in averaging ratios, rates and percentages.
   (a) A.M    (b) G.M    (c) H.M    (d) none
50. G.M is useful in construction of index number.
   (a) true    (b) false    (c) both    (d) none
51. More laborious numerical calculations involves in G.M than A.M
   (a) True    (b) false    (c) both    (d) none
52. H.M is defined when no observation is
   (a) 3    (b) 2    (c) 1    (d) 0
53. When all values occur with equal frequency, there is no
   (a) mode    (b) mean    (c) median    (d) none
54. ________ cannot be treated algebraically
   (a) mode    (b) mean    (c) median    (d) none
55. For the calculation of ________, the data must be arranged in the form of a frequency
   distribution.
   (a) median    (b) mode    (c) mean    (d) none

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MEASURES OF CENTRAL TENDENCY AND DISPERSION

56. ________ is equal to the value corresponding to cumulative frequency
   (a) mode  (b) mean  (c) median  (d) none

57. ________ is the value of the variable corresponding to the highest frequency
   (a) mode  (b) mean  (c) median  (d) none

58. The class in which mode belongs is known as
   (a) median class  (b) mean class  (c) modal class  (d) none

59. The formula of mode is applicable if classes are of ________ width.
   (a) equal  (b) unequal  (c) both  (d) none

60. For calculation of ______ we have to construct cumulative frequency distribution
   (a) mode  (b) median  (c) mean  (d) none

61. For calculation of ______ we have to construct a grouped frequency distribution
   (a) median  (b) mode  (c) mean  (d) none

62. Relation between mean, median & mode is
   (a) mean - mode = 2 (mean - median)  (b) mean - median = 3 (mean - mode)
   (c) mean - median = 2 (mean - mode)  (d) mean - mode = 3 (mean - median)

63. When the distribution is symmetrical, mean, median and mode
   (a) coincide  (b) do not coincide  (c) both  (d) none

64. Mean, median & mode are equal for the
   (a) Binomial distribution  (b) Normal distribution
   (c) both  (d) none

65. In most frequency distributions, it has been observed that the three measures of central
tendency viz. mean, median & mode, obey the approximate relation, provided the
distribution is
   (a) very skew  (b) not very skew  (c) both  (d) none

66. ________ divides the total number of observations into two equal parts.
   (a) mode  (b) mean  (c) median  (d) none

67. Measures which are used to divide or partition the observations into a fixed number of
   parts are collectively known as
   (a) partition values  (b) quartiles  (c) both  (d) none

68. The middle most value of a set of observations is
   (a) median  (b) mode  (c) mean  (d) none

69. The number of observations smaller than ______ is the same as the number larger than it.
   (a) median  (b) mode  (c) mean  (d) none

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70. _______ is the value of the variable corresponding to cumulative frequency $N/2$
   (a) mode   (b) mean   (c) median   (d) none

71. __________ divide the total no. observations into 4 equal parts.
   (a) median   (b) deciles   (c) quartiles   (d) percentiles

72. __________ quartile is known as Upper quartile
   (a) First   (b) Second   (c) Third   (d) none

73. Lower quartile is
   (a) first quartile   (b) second quartile   (c) upper quartile   (d) none

74. The number of observations smaller than lower quartile is the same as the no. lying between lower and middle quartile.
   (a) true   (b) false   (c) both   (d) none

75. _______ are used for measuring central tendency, dispersion & skewness.
   (a) Median   (b) Deciles   (c) Percentiles   (d) Quartiles.

76. The second quartile is known as
   (a) median   (b) lower quartile   (c) upper quartile   (d) none

77. The lower & upper quartiles are used to define
   (a) standard deviation   (b) quartile deviation
   (c) both   (d) none

78. Three quartiles are used in
   (a) Pearson’s formula   (b) Bowley’s formula
   (c) both   (d) none

79. Less than First quartile, the frequency is equal to
   (a) $N/4$   (b) $3N/4$   (c) $N/2$   (d) none

80. Between first & second quartile, the frequency is equal to
   (a) $3N/4$   (b) $N/2$   (c) $N/4$   (d) none

81. Between second & upper quartile, the frequency is equal to
   (a) $3N/4$   (b) $N/4$   (c) $N/2$   (d) none

82. Above upper quartile, the frequency is equal to
   (a) $N/4$   (b) $N/2$   (c) $3N/4$   (d) none

83. Corresponding to first quartile, the cumulative frequency is
   (a) $N/2$   (b) $N/4$   (c) $3N/4$   (d) none
84. Corresponding to second quartile, the cumulative frequency is
   (a) N/4  (b) 2 N/4  (c) 3N/4  (d) none

85. Corresponding to upper quartile, the cumulative frequency is
   (a) 3N/4  (b) N/4  (c) 2N/4  (d) none

86. The values which divide the total number of observations into 10 equal parts are
   (a) quartiles  (b) percentiles  (c) deciles  (d) none

87. There are ———— deciles.
   (a) 7  (b) 8  (c) 9  (d) 10

88. Corresponding to first decile, the cumulative frequency is
   (a) N/10  (b) 2N/10  (c) 9N/10  (d) none

89. Fifth decile is equal to
   (a) mode  (b) median  (c) mean  (d) none

90. The values which divide the total number of observations into 100 equal parts is
   (a) percentiles  (b) quartiles  (c) deciles  (d) none

91. Corresponding to second decile, the cumulative frequency is
   (a) N/10  (b) 2N/10  (c) 5N/10  (d) none

92. There are ———— percentiles.
   (a) 100  (b) 98  (c) 97  (d) 99

93. 10th percentile is equal to
   (a) 1st decile  (b) 10th decile  (c) 9th decile  (d) none

94. 50th percentile is known as
   (a) 50th decile  (b) 50th quartile  (c) mode  (d) median

95. 20th percentile is equal to
   (a) 19th decile  (b) 20th decile  (c) 2nd decile  (d) none

96. (3rd quartile —— 1st quartile)/2 is
   (a) skewness  (b) median  (c) quartile deviation  (d) none

97. 1st percentile is less than 2nd percentile.
   (a) true  (b) false  (c) both  (d) none

98. 25th percentile is equal to
   (a) 1st quartile  (b) 25th quartile  (c) 24th quartile  (d) none

99. 90th percentile is equal to
   (a) 9th quartile  (b) 90th decile  (c) 9th decile  (d) none

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100. 1st decile is greater than 2nd decile
   (a) True       (b) false       (c) both       (d) none

101. Quartile deviation is a measure of dispersion.
   (a) true       (b) false       (c) both       (d) none

102. To define quartile deviation we use
   (a) lower & middle quartiles  (b) lower & upper quartiles
   (c) upper & middle quartiles  (d) none

103. Calculation of quartiles, deciles, percentiles may be obtained graphically from
   (a) Frequency Polygon   (b) Histogram   (c) Ogive   (d) none

104. 7th decile is the abscissa of that point on the Ogive whose ordinate is
   (a) 7N/10       (b) 8N/10       (c) 6N/10       (d) none

105. Rank of median is
   (a) (n+1)/2   (b) (n+1)/4   (c) 3(n+1)/4   (d) none

106. Rank of 1st quartile is
   (a) (n+1)/2   (b) (n+1)/4   (c) 3(n+1)/4   (d) none

107. Rank of 3rd quartile is
   (a) 3(n+1)/4   (b) (n+1)/4   (c) (n+1)/2   (d) none

108. Rank of k th decile is
   (a) (n+1)/2   (b) (n+1)/4   (c) (n+1)/10   (d) k(n+1)/10

109. Rank of k th percentile is
   (a) (n+1)/100  (b) k(n+1)/10  (c) k(n+1)/100  (d) none

110. ———— is equal to value corresponding to cumulative frequency \((N+1)/2\) from simple frequency distribution
    (a) Median   (b) 1st quartile  (c) 3rd quartile  (d) 4th quartile

111. ———— is equal to the value corresponding to cumulative frequency \((N+1)/4\) from simple frequency distribution
    (a) Median   (b) 1st quartile  (c) 3rd quartile  (d) 1st decile

112. ———— is equal to the value corresponding to cumulative frequency 3 \((N+1)/4\) from simple frequency distribution
    (a) Median   (b) 1st quartile  (c) 3rd quartile  (d) 1st decile

113. ———— is equal to the value corresponding to cumulative frequency \(k(N+1)/10\) from simple frequency distribution
    (a) Median   (b) kth decile   (c) kth percentile  (d) none
114. ———— is equal to the value corresponding to cumulative frequency \(k(N + 1)/100\) from simple frequency distribution
(a) \(k^{th}\) decile (b) \(k^{th}\) percentile (c) both (d) none

115. For grouped frequency distribution ———— is equal to the value corresponding to cumulative frequency \(N/2\)
(a) median (b) 1st quartile (c) 3rd quartile (d) none

116. For grouped frequency distribution ———— is equal to the value corresponding to cumulative frequency \(N/4\)
(a) median (b) 1st quartile (c) 3rd quartile (d) none

117. For grouped frequency distribution ———— is equal to the value corresponding to cumulative frequency \(3N/4\)
(a) median (b) 1st quartile (c) 3rd quartile (d) none

118. For grouped frequency distribution ———— is equal to the value corresponding to cumulative frequency \(kN/10\)
(a) median (b) \(k^{th}\) percentile (c) \(k^{th}\) decile (d) none

119. For grouped frequency distribution ———— is equal to the value corresponding to cumulative frequency \(kN/100\)
(a) \(k^{th}\) quartile (b) \(k^{th}\) percentile (c) \(k^{th}\) decile (d) none

120. In Ogive, abscissa corresponding to ordinate \(N/2\) is
(a) median (b) 1st quartile (c) 3rd quartile (d) none

121. In Ogive, abscissa corresponding to ordinate \(N/4\) is
(a) median (b) 1st quartile (c) 3rd quartile (d) none

122. In Ogive, abscissa corresponding to ordinate \(3N/4\) is
(a) median (b) 3rd quartile (c) 1st quartile (d) none

123. In Ogive, abscissa corresponding to ordinate ———— is \(k^{th}\) decile.
(a) \(kN/10\) (b) \(kN/100\) (c) \(kN/50\) (d) none

124. In Ogive, abscissa corresponding to ordinate ———— is \(k^{th}\) percentile.
(a) \(kN/10\) (b) \(kN/100\) (c) \(kN/50\) (d) none

125. For 899, 999, 391, 384, 590, 480, 485, 760, 111, 240
Rank of median is
(a) 2.75 (b) 5.5 (c) 8.25 (d) none

126. For 333, 999, 888, 777, 666, 555, 444
Rank of 1st quartile is
(a) 3 (b) 1 (c) 2 (d) 7
127. For 333, 999, 888, 777, 1000, 321, 133
    Rank of 3rd quartile is
    (a) 7  (b) 4  (c) 5  (d) 6

128. Price per kg (₹) : 45 50 35; Kgs. Purchased : 100 40 60 Total frequency is
    (a) 300  (b) 100  (c) 150  (d) 200

129. The length of a rod is measured by a tape 10 times. You are to estimate the length of the rod by averaging these 10 determinations.
    What is the suitable form of average in this case?
    (a) A.M  (b) G.M  (c) H.M  (d) none

130. A person purchases 5 rupees worth of eggs from 10 different markets. You are to find the average no. of eggs per rupee for all the markets taken together. What is the suitable form of average in this case?
    (a) A.M  (b) G.M  (c) H.M  (d) none

131. You are given the population of India for the courses of 1981 & 1991. You are to find the population of India at the middle of the period by averaging these population figures, assuming a constant rate of increase of population.
    What is the suitable form of average in this case?
    (a) A.M  (b) G.M  (c) H.M  (d) none

132. ———— is least affected by sampling fluctuations.
    (a) Standard deviation  (b) Quartile deviation  (c) both  (d) none

133. “Root–Mean Square Deviation from Mean” is
    (a) Standard deviation  (b) Quartile deviation  (c) both  (d) none

134. Standard Deviation is
    (a) absolute measure  (b) relative measure  (c) both  (d) none

135. Coefficient of variation is
    (a) absolute measure  (b) relative measure  (c) both  (d) none

136. ———— deviation is called semi-interquartile range.
    (a) Percentile  (b) Standard  (c) Quartile  (d) none

137. ———— Deviation is defined as half the difference between the lower & upper quartiles.
    (a) Quartile  (b) Standard  (c) both  (d) none
138. Quartile Deviation for the data 1, 3, 4, 5, 6, 6, 10 is
   (a) 3  (b) 1  (c) 6  (d) 1.5

139. Coefficient of Quartile Deviation is
   (a) \(\frac{\text{Quartile Deviation} \times 100}{\text{Median}}\)
   (b) \(\frac{\text{Quartile Deviation} \times 100}{\text{Mean}}\)
   (c) \(\frac{\text{Quartile Deviation} \times 100}{\text{Mode}}\)
   (d) none

140. Mean for the data 6, 4, 1, 6, 5, 10, 3 is
   (a) 7  (b) 5  (c) 6  (d) none

141. Coefficient of variation = \(\frac{\text{Standard Deviation} \times 100}{\text{Mean}}\)
   (a) true  (b) false  (c) both  (d) none

142. If mean = 5, Standard deviation = 2.6 then the coefficient of variation is
   (a) 49  (b) 51  (c) 50  (d) 52

143. If median = 5, Quartile deviation = 1.5 then the coefficient of quartile deviation is
   (a) 33  (b) 35  (c) 30  (d) 20

144. A.M of 2, 6, 4, 1, 8, 5, 2 is
   (a) 4  (b) 3  (c) 4  (d) none

145. Most useful among all measures of dispersion is
   (a) S.D  (b) Q.D  (c) Mean deviation  (d) none

146. For the observations 6, 4, 1, 6, 5, 10, 4, 8 Range is
   (a) 10  (b) 9  (c) 8  (d) none

147. A measure of central tendency tries to estimate the
   (a) central value  (b) lower value  (c) upper value  (d) none

148. Measures of central tendency are known as
   (a) differences  (b) averages  (c) both  (d) none

149. Mean is influenced by extreme values.
   (a) true  (b) false  (c) both  (d) none

150. Mean of 6, 7, 11, 8 is
    (a) 11  (b) 6  (c) 7  (d) 8

151. The sum of differences between the actual values and the arithmetic mean is
    (a) 2  (b) -1  (c) 0  (d) 1

152. When the algebraic sum of deviations from the arithmetic mean is not equal to zero, the
    figure of arithmetic mean ———— correct.
    (a) is  (b) is not  (c) both  (d) none

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153. In the problem
No. of persons: 15 14 42 27 18
The assumed mean is
(a) 34 (b) 37 (c) 40 (d) 43

154. In the problem
Size of items: 1–3 3–8 8–15 15–26
Frequency: 5 10 16 15
The assumed mean is
(a) 20.5 (b) 2 (c) 11.5 (d) 5.5

155. The average of a series of overlapping averages, each of which is based on a certain number of item within a series is known as
(a) moving average (b) weighted average (c) simple average (d) none

156. ———— averages is used for smoothening a time series.
(a) moving average (b) weighted average (c) simple average (d) none

157. Pooled Mean is also called
(a) Mean (b) Geometric Mean (c) Grouped Mean (d) none

158. Half of the numbers in an ordered set have values less than the ———— and half will have values greater than the ———— .
(a) mean, median (b) median, median (c) mode, mean (d) none.

159. The median of 27, 30, 26, 44, 42, 51, 37 is
(a) 30 (b) 42 (c) 44 (d) 37

160. For an even number of values the median is the
(a) average of two middle values (b) middle value (c) both (d) none

161. In the case of a continuous frequency distribution, the size of the ———— item indicates class interval in which the median lies.
(a) (n-1)/2th (b) (n + 1)/2th (c) n/2th (d) none

162. The deviations from median are ———— if negative signs are ignored as compared to other measures of central tendency.
(a) minimum (b) maximum (c) same (d) none
163. Ninth Decile lies in the class interval of the item
   (a) $n/9$  (b) $9n/10$  (c) $9n/20$  (d) none item.

164. Ninety Ninth Percentile lies in the class interval of the item
   (a) $99n/100$  (b) $99n/10$  (c) $99n/200$  (d) none item.

165. ________ is the value of the variable at which the concentration of observation is the densest.
   (a) mean  (b) median  (c) mode  (d) none

   No. of students:  15  118  142  127  18
   Modal group is
   (a) 66–68  (b) 69–71  (c) 63–65  (d) none

167. A distribution is said to be symmetrical when the frequency rises & falls from the highest value in the __________ proportion.
   (a) unequal  (b) equal  (c) both  (d) none

168. ________ always lies in between the arithmetic mean & mode.
   (a) G.M  (b) H.M  (c) Median  (d) none

169. Logarithm of G.M is the ________ of logarithms of the different values.
   (a) weighted mean  (b) simple mean  (c) both  (d) none

170. ________ is not much affected by fluctuations of sampling.
   (a) A.M  (b) G.M  (c) H.M  (d) none

171. The data 1, 2, 4, 8, 16 are in
   (a) Arithmetic progression  (b) Geometric progression
   (c) Harmonic progression  (d) none

172. ________ & ________ can not be calculated if any observation is zero.
   (a) G.M & A.M  (b) H.M & A.M  (c) H.M & G. M  (d) None.

173. ________ & ________ are called ratio averages.
   (a) H.M & G.M  (b) H.M & A.M  (c) A.M & G.M  (d) none

174. ________ is a good substitute to a weighted average.
   (a) A.M  (b) G.M  (c) H.M  (d) none

175. For ordering shoes of various sizes for resale, a ________ size will be more appropriate.
   (a) median  (b) modal  (c) mean  (d) none

176. ________ is called a positional measure.
   (a) mean  (b) mode  (c) median  (d) none
177. 50% of actual values will be below & 50% of will be above ————
   (a) mode    (b) median    (c) mean    (d) none

178. Extreme values have ———— effect on mode.
   (a) high    (b) low    (c) no    (d) none

179. Extreme values have ———— effect on median.
   (a) high    (b) low    (c) no    (d) none

180. Extreme values have ———— effect on A.M.
   (a) greatest    (b) least    (c) some    (d) none

181. Extreme values have ———— effect on H.M.
   (a) least    (b) greatest    (c) medium    (d) none

182. ———— is used when representation value is required & distribution is asymmetric.
   (a) mode    (b) mean    (c) median    (d) none

183. ———— is used when most frequently occurring value is required (discrete variables).
   (a) mode    (b) mean    (c) median    (d) none

184. ———— is used when rate of growth or decline required.
   (a) mode    (b) A.M    (c) G.M    (d) none

185. In finding ————, the distribution has open-end classes.
   (a) median    (b) mean    (c) standard deviation    (d) none

186. The cumulative frequency distribution is used for
   (a) median    (b) mode    (c) mean    (d) none

187. In ———— the quantities are in ratios.
   (a) A.M    (b) G.M    (c) H.M    (d) none

188. ———— is used when variability has also to be calculated.
   (a) A.M    (b) G.M    (c) H.M    (d) none

189. ———— is used when the sum of absolute deviations from the average should be least.
   (a) Mean    (b) Mode    (c) Median    (d) None

190. ———— is used when sampling variability should be least.
   (a) Mode    (b) Median    (c) Mean    (d) none

191. ———— is used when distribution pattern has to be studied at varying levels.
   (a) A.M    (b) Median    (c) G.M    (d) none
192. The average discovers
(a) uniformity in variability
(b) variability in uniformity of distribution
(c) both
(d) none

193. The average has relevance for
(a) homogeneous population
(b) heterogeneous population
(c) both
(d) none

194. The correction factor is applied in
(a) inclusive type of distribution
(b) exclusive type of distribution
(c) both
(d) none

195. “Mean has the least sampling variability” prove the mathematical property of mean
(a) True
(b) false
(c) both
(d) none

196. “The sum of deviations from the mean is zero” —— is the mathematical property of mean
(a) True
(b) false
(c) both
(d) none

197. “The mean of the two samples can be combined” — is the mathematical property of mean
(a) True
(b) false
(c) both
(d) none

198. “Choices of assumed mean does not affect the actual mean” — prove the mathematical property of mean
(a) True
(b) false
(c) both
(d) none

199. “In a moderately asymmetric distribution mean can be found out from the given values of median & mode” — is the mathematical property of mean
(a) True
(b) false
(c) both
(d) none

200. The mean wages of two companies are equal. It signifies that the workers of both the companies are equally well-off.
(a) True
(b) false
(c) both
(d) none

201. The mean wage in factory A is Rs. 6,000 whereas in factory B it is Rs. 5,500. It signifies that factory A pays more to all its workers than factory B.
(a) True
(b) false
(c) both
(d) none

202. Mean of 0, 3, 5, 6, 7, 9, 12, 0, 2 is
(a) 4.9
(b) 5.7
(c) 5.6
(d) none

203. Median of 15, 12, 6, 13, 12, 15, 8, 9 is
(a) 13
(b) 8
(c) 12
(d) 9

204. Median of 0.3, 5, 6, 7, 9, 12, 0, 2 is
(a) 7
(b) 6
(c) 3
(d) 5
205. Mode of 0, 3, 5, 6, 7, 9, 12, 0, 2 is
   (a) 6  (b) 0  (c) 3  (d) 5
206. Mode of 15, 12, 5, 13, 12, 15, 8, 8, 9, 10, 15 is
   (a) 15  (b) 12  (c) 8  (d) 9
207. Median of 40, 50, 30, 20, 25, 35, 30, 30, 20, 30 is
   (a) 25  (b) 30  (c) 35  (d) none
208. Mode of 40, 50, 30, 20, 25, 35, 30, 30, 20, 30 is
   (a) 25  (b) 30  (c) 35  (d) none
209. ________ in particular helps in finding out the variability of the data.
   (a) Dispersion  (b) Median  (c) Mode  (d) None
210. Measures of central tendency are called averages of the ________ order.
    (a) 1st  (b) 2nd  (c) 3rd  (d) none
211. Measures of dispersion are called averages of the ________ order.
    (a) 1st  (b) 2nd  (c) 3rd  (d) none
212. In measuring dispersion, it is necessary to know the amount of ________ & the degree of ________.
    (a) variation, variation  (b) variation, median  
    (c) median, variation  (d) none
213. The amount of variation is designated as ________ measure of dispersion.
    (a) relative  (b) absolute  (c) both  (d) none
214. The degree of variation is designated as ________ measure of dispersion.
    (a) relative  (b) absolute  (c) both  (d) none
215. For purposes of comparison between two or more series with varying size or no. of items, 
    varying central values or units of calculation, only ________ measures can be used.
    (a) absolute  (b) relative  (c) both  (d) none
216. The relation Relative range = Absolute range/Sum of the two extremes. is
    (a) True  (b) false  (c) both  (d) none
217. The relation Absolute range = Relative range/Sum of the two extremes is 
    (a) True  (b) false  (c) both  (d) none
218. In quality control ________ is used as a substitute for standard deviation.
    (a) mean deviation  (b) median  (c) range  (d) none
219. ________ factor helps to know the value of standard deviation.
    (a) Correction  (b) Range  (c) both  (d) none
220. ————- is extremely sensitive to the size of the sample
   (a) Range  (b) Mean  (c) Median  (d) Mode
221. As the sample size increases, ————- also tends to increase.
   (a) Range  (b) Mean  (c) Median  (d) Mode
222. As the sample size increases, range also tends to increase though not proportionately.
   (a) true  (b) false  (c) both  (d) none.
223. As the sample size increases, range also tends to
   (a) decrease  (b) increase  (c) same  (d) none
224. The dependence of range on extreme items can be avoided by adopting
   (a) standard deviation  (b) mean deviation  (c) quartile deviation  (d) none
225. Quartile deviation is called
   (a) semi inter quartile range  (b) quartile range  (c) both  (d) none
226. When 1st quartile = 20, 3rd quartile = 30, the value of quartile deviation is
   (a) 7  (b) 4  (c) -5  (d) 5
227. \((Q_3 - Q_1)/(Q_3 + Q_1)\) is
   (a) coefficient of Quartile Deviation  (b) coefficient of Mean Deviation
   (c) coefficient of Standard deviation  (d) none
228. Standard deviation is denoted by
   (a) \(\sigma^2\)  (b) \(\sigma\)  (c) \(\sqrt{\sigma}\)  (d) none
229. The square of standard deviation is known as
   (a) variance  (b) standard deviation
   (c) mean deviation  (d) none
230. Mean of 25, 32, 43, 53, 62, 59, 48, 31, 24, 33 is
   (a) 44  (b) 43  (c) 42  (d) 41
231. For the following frequency distribution
   Class interval: 10–20  20–30  30–40  40–50  50–60  60–70
   Frequency: 20  9  31  18  10  9
   assumed mean can be taken as
   (a) 55  (b) 45  (c) 35  (d) none
232. The value of the standard deviation does not depend upon the choice of the origin.
   (a) True  (b) false  (c) both  (d) none
233. Coefficient of standard deviation is
   (a) S.D/Median  (b) S.D/Mean  (c) S.D/Mode  (d) none

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234. The value of the standard deviation will change if any one of the observations is changed.
   (a) True  (b) false  (c) both  (d) none

235. When all the values are equal then variance & standard deviation would be
   (a) 2  (b) -1  (c) 1  (d) 0

236. For values lie close to the mean, the standard deviations are
   (a) big  (b) small  (c) moderate  (d) none

237. If the same amount is added to or subtracted from all the values, variance & standard deviation shall
   (a) changed  (b) unchanged  (c) both  (d) none

238. If the same amount is added to or subtracted from all the values, the mean shall increase or decrease by the ———— amount
   (a) big  (b) small  (c) same  (d) none

239. If all the values are multiplied by the same quantity, the ———— & ———— also would be multiple of the same quantity.
   (a) mean, standard deviation  (b) mean, median
   (c) mean, mode  (d) median, deviations

240. For a moderately non-symmetrical distribution, Mean deviation = 4/5 of standard deviation
   (a) true  (b) false  (c) both  (d) none

241. For a moderately non-symmetrical distribution, Quartile deviation = Standard deviation/3
   (a) true  (b) false  (c) both  (d) none

242. For a moderately non-symmetrical distribution, probable error of standard deviation = Standard deviation/3
   (a) true  (b) false  (c) both  (d) none

243. Quartile deviation = Probable error of Standard deviation.
   (a) true  (b) false  (c) both  (d) none

244. Coefficient of Mean Deviation is
   (a) Mean deviation x 100/Mean or mode  (b) Standard deviation x 100/Mean or median
   (c) Mean deviation x 100/Mode or median  (d) none

245. Coefficient of Quartile Deviation = Quartile Deviation x 100/Median
   (a) true  (b) false  (c) both  (d) none

246. Karl Pearson’s measure gives
   (a) coefficient of Mean Variation  (b) coefficient of Standard deviation
   (c) coefficient of variation  (d) none
247. In ——— range has the greatest use.
   (a) Time series    (b) quality control    (c) both    (d) none

248. Mean is an absolute measure & standard deviation is based upon it. Therefore standard deviation is a relative measure.
   (a) true    (b) false    (c) both    (d) none

249. Semi-quartile range is one-fourth of the range in a normal symmetrical distribution.
   (a) Yes    (b) No    (c) both    (d) none

250. Whole frequency table is needed for the calculation of
   (a) range    (b) variance    (c) both    (d) none

251. Relative measures of dispersion make deviations in similar units comparable.
   (a) true    (b) false    (c) both    (d) none

252. Quartile deviation is based on the
   (a) highest 50%    (b) lowest 25%
   (c) highest 25%    (d) middle 50% of the item.

253. S.D is less than Mean deviation
   (a) true    (b) false    (c) both    (d) none

254. Coefficient of variation is independent of the unit of measurement.
   (a) true    (b) false    (c) both    (d) none

255. Coefficient of variation is a relative measure of
   (a) mean    (b) deviation    (c) range    (d) dispersion.

256. Coefficient of variation is equal to
   (a) Standard deviation x 100 / median    (b) Standard deviation x 100 / mode
   (c) Standard deviation x 100 / mean    (d) none

257. Coefficient of Quartile Deviation is equal to
   (a) Quartile deviation x 100 / median    (b) Quartile deviation x 100 / mean
   (c) Quartile deviation x 100 / mode    (d) none

258. If each item is reduced by 15 A.M is
   (a) reduced by 15    (b) increased by 15    (c) reduced by 10    (d) none

259. If each item is reduced by 10, the range is
   (a) increased by 10    (b) decreased by 10    (c) unchanged    (d) none

260. If each item is reduced by 20, the standard deviation
   (a) increased    (b) decreased    (c) unchanged    (d) none
261. If the variables are increased or decreased by the same amount the standard deviation is
(a) decreased  (b) increased  (c) unchanged  (d) none

262. If the variables are increased or decreased by the same proportion, the standard deviation changes by
(a) same proportion  (b) different proportion  (c) both  (d) none

263. The mean of the 1st n natural no. is
(a) n/2  (b) (n-1)/2  (c) (n+1)/2  (d) none

264. If the class interval is open-end then it is difficult to find
(a) frequency  (b) A.M  (c) both  (d) none

265. Which one is true—
(a) A.M = assumed mean + arithmetic mean of deviations of terms
(b) G.M = assumed mean + arithmetic mean of deviations of terms
(c) Both  (d) none

266. If the A.M of any distribution be 25 & one term is 18. Then the deviation of 18 from A.M is
(a) 7  (b) -7  (c) 43  (d) none

267. For finding A.M in Step-deviation method, the class intervals should be of
(a) equal lengths  (b) unequal lengths  (c) maximum lengths  (d) none

268. The sum of the squares of the deviations of the variable is ———— when taken about A.M
(a) maximum  (b) zero  (c) minimum  (d) none

269. The A.M of 1, 3, 5, 6, x, 10 is 6. The value of x is
(a) 10  (b) 11  (c) 12  (d) none

270. The G.M of 2 & 8 is
(a) 2  (b) 4  (c) 8  (d) none

271. (n+1)/2 th term is median if n is
(a) odd  (b) even  (c) both  (d) none

272. For the values of a variable 5, 2, 8, 3, 7, 4, the median is
(a) 4  (b) 4.5  (c) 5  (d) none

273. The abscissa of the maximum frequency in the frequency curve is the
(a) mean  (b) median  (c) mode  (d) none

274. Variable: 2 3 4 5 6 7 8 13 7 4
   No. of men: 5 6 8 13 7 4
   Mode is
(a) 6  (b) 4  (c) 5  (d) none
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275. The class having maximum frequency is called
(a) modal class    (b) median class    (c) mean class    (d) none

276. For determination of mode, the class intervals should be
(a) overlapping    (b) maximum    (c) minimum    (d) none

277. First Quartile lies in the class interval of the
(a) \(\frac{n}{2}\)th item    (b) \(\frac{n}{4}\)th item    (c) \(\frac{3n}{4}\)th item    (d) \(\frac{n}{10}\)th item

278. The value of a variate that occur most often is called
(a) median    (b) mean    (c) mode    (d) none

279. For the values of a variable 3, 1, 5, 2, 6, 8, 4 the median is
(a) 3    (b) 5    (c) 4    (d) none

280. If \(y = 5x - 20\) & \(\bar{x} = 30\) then the value of \(y\) is
(a) 130    (b) 140    (c) 30    (d) none

281. If \(y = 3x - 100\) and \(\bar{x} = 50\) then the value of \(y\) is
(a) 60    (b) 30    (c) 100    (d) 50

282. The median of the numbers 11, 10, 12, 13, 9 is
(a) 12.5    (b) 12    (c) 10.5    (d) 11

283. The mode of the numbers 7, 7, 7, 9, 10, 11, 11, 11, 12 is
(a) 11    (b) 12    (c) 7    (d) 7 & 11

284. In a symmetrical distribution when the 3rd quartile plus 1st quartile is halved, the value would give
(a) mean    (b) mode    (c) median    (d) none

285. In Zoology ———— is used.
(a) median    (b) mean    (c) mode    (d) none

286. For calculation of Speed & Velocity
(a) G.M    (b) A.M    (c) H.M    (d) none is used.

287. The S.D is always taken from
(a) median    (b) mode    (c) mean    (d) none

288. Coefficient of Standard deviation is equal to
(a) \(\frac{S.D}{A.M}\)    (b) \(\frac{A.M}{S.D}\)    (c) \(\frac{S.D}{G.M}\)    (d) none

289. The distribution, for which the coefficient of variation is less, is ——— consistent.
(a) less    (b) more    (c) moderate    (d) none
# ANSWERS

1. (b) 2. (a) 3. (c) 4. (a) 5. (b)  
6. (a) 7. (d) 8. (c) 9. (b) 10. (a)  
11. (a) 12. (a) 13. (b) 14. (d) 15. (a)  
16. (d) 17. (b) 18. (a) 19. (b) 20. (a)  
21. (b) 22. (d) 23. (a) 24. (c) 25. (b)  
26. (a) 27. (a) 28. (b) 29. (b) 30. (a)  
31. (c) 32. (a) 33. (b) 34. (a) 35. (a)  
36. (a) 37. (a) 38. (a) 39. (a) 40. (a)  
41. (c) 42. (a) 43. (a) 44. (b) 45. (a)  
46. (b) 47. (a) 48. (a) 49. (b) 50. (a)  
51. (a) 52. (d) 53. (a) 54. (a) 55. (b)  
56. (c) 57. (a) 58. (c) 59. (c) 60. (b)  
61. (b) 62. (d) 63. (a) 64. (b) 65. (b)  
66. (c) 67. (c) 68. (a) 69. (a) 70. (c)  
71. (c) 72. (c) 73. (a) 74. (a) 75. (d)  
76. (a) 77. (b) 78. (b) 79. (a) 80. (c)  
81. (b) 82. (a) 83. (b) 84. (b) 85. (a)  
86. (c) 87. (c) 88. (a) 89. (b) 90. (a)  
91. (b) 92. (d) 93. (a) 94. (d) 95. (c)  
96. (c) 97. (a) 98. (a) 99. (c) 100. (b)  
101. (a) 102. (b) 103. (c) 104. (a) 105. (a)  
106. (b) 107. (a) 108. (d) 109. (c) 110. (a)  
111. (b) 112. (c) 113. (b) 114. (b) 115. (a)  
116. (b) 117. (c) 118. (c) 119. (b) 120. (a)  
121. (b) 122. (b) 123. (a) 124. (b) 125. (b)  
126. (c) 127. (d) 128. (d) 129. (a) 130. (c)  
131. (b) 132. (a) 133. (a) 134. (a) 135. (b)  
136. (c) 137. (a) 138. (d) 139. (a) 140. (b)  
141. (a) 142. (d) 143. (c) 144. (c) 145. (a)  
146. (b) 147. (a) 148. (b) 149. (a) 150. (d)  
151. (c) 152. (b) 153. (b) 154. (c) 155. (a)
| 156. (a) | 157. (c) | 158. (b) | 159. (d) | 160. (a) |
| 161. (c) | 162. (a) | 163. (b) | 164. (a) | 165. (c) |
| 166. (a) | 167. (b) | 168. (c) | 169. (a) | 170. (b) |
| 171. (b) | 172. (c) | 173. (c) | 174. (c) | 175. (b) |
| 176. (c) | 177. (b) | 178. (d) | 179. (c) | 180. (c) |
| 181. (b) | 182. (b) | 183. (b) | 184. (c) | 185. (a) |
| 186. (a) | 187. (b) | 188. (a) | 189. (c) | 190. (c) |
| 191. (b) | 192. (a) | 193. (b) | 194. (b) | 195. (b) |
| 196. (a) | 197. (a) | 198. (a) | 199. (b) | 200. (b) |
| 201. (b) | 202. (a) | 203. (c) | 204. (d) | 205. (b) |
| 206. (a) | 207. (b) | 208. (b) | 209. (a) | 210. (a) |
| 211. (b) | 212. (a) | 213. (b) | 214. (a) | 215. (b) |
| 216. (a) | 217. (b) | 218. (b) | 219. (a) | 220. (a) |
| 221. (a) | 222. (a) | 223. (b) | 224. (c) | 225. (a) |
| 226. (d) | 227. (a) | 228. (b) | 229. (a) | 230. (d) |
| 231. (c) | 232. (a) | 233. (b) | 234. (a) | 235. (d) |
| 236. (b) | 237. (b) | 238. (c) | 239. (a) | 240. (b) |
| 241. (b) | 242. (b) | 243. (a) | 244. (c) | 245. (a) |
| 246. (c) | 247. (b) | 248. (b) | 249. (a) | 250. (c) |
| 251. (b) | 252. (d) | 253. (a) | 254. (a) | 255. (d) |
| 256. (c) | 257. (a) | 258. (a) | 259. (c) | 260. (c) |
| 261. (c) | 262. (a) | 263. (c) | 264. (a) | 265. (c) |
| 266. (b) | 267. (a) | 268. (c) | 269. (b) | 270. (b) |
| 271. (a) | 272. (b) | 273. (c) | 274. (c) | 275. (a) |
| 276. (a) | 277. (b) | 278. (c) | 279. (c) | 280. (a) |
| 281. (d) | 282. (d) | 283. (d) | 284. (c) | 285. (c) |
| 286. (c) | 287. (c) | 288. (a) | 289. (b) |